ARCHIVES OF TRANSPORT *ISSN (print):* *0866-9546*
Volume 78, Issue 2, 2026 ***e-ISSN (online):*** *2300-8830*
DOI: *10.61089/aot2026.mts3f726*

TECHNOLOGIES FOR OPTIMAL LOCATION OF LOGISTICS CENTERS FOR CRITICAL INFRASTRUCTURE

Serhii DZIUBA[1], Larysa KORIASHKINA[2], Danylo LUBENETS[3], Boris BLYUSS[4], Yana LITVINOVA[5]

[1] M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine, Dnipro, Ukraine
[2, 3] Department of System Analysis and Control, Dnipro University of Technology, Dnipro, Ukraine
[4] Prydneprovsk Research Center of the National Academy of Sciences of Ukraine and of Ministry of Education and Science of Ukraine, Dnipro, Ukraine
[5] Department of Transport Management, Dnipro University of Technology, Dnipro, Ukraine

Abstract:

The paper is devoted to current issues concerning the development of critical infrastructure and the improvement of the efficiency of its units, particularly in planning proactive organizational and technical measures for rapid response to natural or man-made emergencies. The aim of the study is to create technologies and, based on them, to develop a software system for the placement of critical infrastructure logistics centers, as well as the determination of resource flows between them, taking into account the limited capacities of the centers and ensuring minimal transportation and organizational costs. The technologies developed and presented in the study are based on mathematical tools, including models and methods for optimal facility location and multiplex partitioning of continuous sets, as well as models and methods for two-stage distribution of material flows in hierarchical logistics systems. The created software system, written in Python 3 and integrated with modern geographic information systems, combines the necessary functionality for researching center placement algorithms, determining their service areas, analyzing material flows between structural elements of critical infrastructure, and rationally selecting locations for logistics facilities based on existing ones. The presented results of logistics system modeling confirm the correctness of the algorithms and demonstrate the applicability of the multifunctional software system for supporting managerial decisions related to the development of critical infrastructure, reducing transportation and operational costs through the optimization of material flows between structural units. The use of the Google Maps API cloud platform for obtaining spatial data allows for the rational distribution of material resources across existing or newly optimally placed centers, considering their capabilities and spatio-temporal criteria. In studying two-stage distribution processes, the developed technology supports the formulation of proposals for organizing such processes, aligning the local interests and functional goals of individual logistics system elements with the main objective: achieving optimal overall transportation, time, and associated costs.

Keywords: *critical infrastructure, logistics center, optimization, multistage distribution, multiplex zoning, facility location, geoinformation technologies*

To cite this article:

Dziuba, S., Koriashkina, L., Lubenets, D., Blyuss, B., & Litvinova, Y. (2026). Technologies for optimal location of logistics centers for critical infrastructure. Archives of Transport, 78(2), 59-73. https://doi.org/10.61089/aot2026.mts3f726



Contact:
1) sergejdzuba@gmail.com [https://orcid.org/0000-0002-3139-2989]; 2) koriashkina.l.s@nmu.one [https://orcid.org/0000-0001-6423-092X]; 3) lubenets.d.y@nmu.one [https://orcid.org/0009-0000-8563-3760]; 4) BOBlus1952@nas.gov.ua [https://orcid.org/0000-0003-3077-1220] – corresponding author; 5) litvinova.ya.v@nmu.one [https://orcid.org/0000-0003-2806-4076]

Article is available in open access and licensed under a Creative Commons Attribution 4.0 International (CC BY 4.0)

1. Introduction

Since the 2000s, there has been a growing interest within the scientific community in studying the transformation of critical infrastructure in the context of the digital economy, as evidenced by the analysis of publication activity conducted by Trushkina (2024). A significant number of research papers over the past quarter of the current century have been devoted to exploring strategies for the effective development of critical infrastructure under conditions of digital transformation.

The digital transformation of critical infrastructure is a key component of the broader strategy for reshaping the global economy. Properly selected digital technologies, combined with employee competencies, serve as the foundation for critical infrastructure entities to rapidly adapt to crisis situations, modernize work processes, meet ever-changing customer needs, stimulate and implement innovative managerial solutions. However, as the author emphasizes, existing methodologies for assessing the development of critical infrastructure remain imperfect and do not fully correspond to modern global challenges.

A comprehensive literature review on the application of Industry 4.0 technologies in logistics processes and their impact on economic, environmental, and social sustainability was conducted by Ferraro et al. (2023). The authors recognize the significant influence of Industry 4.0 technologies on logistics.

New business opportunities associated with Industry 4.0 and company adaptation to the evolving environment are highlighted by Rubio et al. (2021). They propose a multi-objective optimization algorithm aimed at improving productivity and reducing costs and energy consumption in autonomous industrial processes in order to achieve sustainable development.

With the increasing number of natural disasters, technological accidents, and military conflicts in recent years, the need to ensure the protection and reliable maintenance of critical infrastructure has grown significantly. The uninterrupted functioning of critical infrastructure is an essential component of the sustainable development of a country as a whole. This study is devoted to the development of computer technologies for the optimal location of logistics centers of critical infrastructure in order to enhance the effectiveness of managerial decision-making concerning the rational territorial allocation of service centers and the distribution of material flows, particularly in the context of man-made emergencies. Rigorous mathematical justifications of models and methods for optimal location of centers, along with the simultaneous determination of service zones, are presented in the works of Dziuba et al. (2023) and Koriashkina et al. (2025). The technological solutions for the location of technical support facilities for critical infrastructure developed and presented in this study are based precisely on this mathematical foundation. Unlike well-known location–allocation problems, the models considered here account for the possibility of servicing facilities by multiple nearby service centers, the multistage nature of distribution processes (Koriashkina et al., 2024), and the option of selecting locations for centers based on existing facilities (Dziuba et al., 2023).

2. Literature Review

Scientific publications of the last 5–7 years related to the optimal location of logistics facilities can be grouped into three interrelated streams: covering-based facility location models, models of hierarchical (multistage) logistics systems with resource distribution, and generalized optimization frameworks together with the solution techniques that support them.

The first stream develops the set covering problem in continuous or discrete formulations, which remains one of the principal methods of spatial analysis in support of location decision-making. Within it, researchers have addressed the selection of network hubs for routing flows in networks expanding under strategic and operational uncertainty (Escudero & Monge, 2021), the minimization of customer service time using geoinformation services (Delman et al., 2016), the rational selection of service center locations with overlapping service zones that preserve coverage when a particular center is unable to serve its customers (Dziuba et al., 2023), multi-period maximal covering with capacitated modular facilities that can be relocated as demand changes (Alizadeh et al., 2021), the joint optimization of facility allocation and maintenance over the network life cycle (Wu et al., 2021), and the balancing of facility workloads under maximal demand coverage (Xu et al., 2023). The ranking of candidate sites by multi-criteria decision-making techniques such as AHP and TOPSIS complements these models on the practical side (Stopka et al., 2022). Taken together, these studies reveal a clear tendency to enrich covering models with

capacity constraints, uncertainty, and reliability (multiple-coverage) considerations; at the same time, accounting for such factors substantially increases computational effort, forcing most authors to resort to heuristic procedures or bounds on the objective function instead of exact methods.

The second stream concerns hierarchical logistics systems and the distribution of material resources within them. One group of works focuses on the design of multi-level networks: hub-and-spoke structures under capacity constraints and the risk of deliberate interdiction (Bansal et al., 2024), exact identification of globally optimal facility locations in continuous space (Gao et al., 2021), and multi-level maintenance networks in agriculture, formulated either as mixed-integer programs with hierarchical and adjacency constraints (Li J. et al., 2023) or as set covering models solved by an improved genetic algorithm (Wang et al., 2021). Simulation-based analyses further show how warehouse location decisions propagate to the efficiency of the entire supply chain under stochastic demand (Szczepański et al., 2019).

Another group of works addresses resource distribution under emergency and uncertainty conditions: two-stage location–allocation models for evacuation and primary aid provision (Koriashkina et al., 2024) and their special case with fixed centers (Kiseleva et al., 2020), stochastic network flow models that allow relief centers to support each other (Chang et al., 2022), reliable location–allocation of emergency service stations under maintenance uncertainty (Lu et al., 2019), joint location–allocation–inventory planning with epistemic uncertainty in demand (Li P. et al., 2023), and two-stage medical logistics problems solved by a modified genetic algorithm (Serhieiev & Us, 2023). A two-stage decomposition has also been applied to the design of multi-level low-carbon distribution networks, where transit points and flows are selected at the first stage and delivery routes at the second (Fu et al., 2024). Related sector-specific studies optimize the placement of distributed generation in power distribution networks (Pirouzi et al., 2024), develop forecasting toolkits for the functioning of critical energy infrastructure under volatile security environments (Terentiev et al., 2023), and justify the choice of effective freight transportation technologies (Tsopa et al., 2025). Common to these works are the minimization of total costs across the levels of the hierarchy, the explicit treatment of uncertainty, and the reliance on metaheuristics or decomposition schemes owing to the complexity of the resulting models.

The third stream seeks unification and methodological support. Ordered median optimization generalizes the classical p-median and p-center problems and makes it possible to incorporate equity and complex customer preference structures into facility location (Ljubić et al., 2024), including its capacitated GIS-based vector assignment version (Vafaeinejad et al., 2020). Advances in mixed-integer linear programming solvers underpin the practical solvability of most of the models mentioned above (Clautiaux & Ljubić, 2025), while reviews of modern trends in logistics development (Zagurskiy, 2021) and of intermodal hub network design (Basallo-Triana et al., 2021) outline research avenues for modeling realistic transport systems.

The synthesis of these studies shows that continuously distributed demand, multiple (redundant) coverage of facilities by several nearest centers, capacity constraints, and the multi-stage organization of resource flows are, as a rule, considered separately; a technology combining them within a single mathematical and software framework integrated with geoinformation systems, as required for the logistics of critical infrastructure, has not yet been proposed. Filling this gap is the focus of the present study.

The present study investigates the process of locating technical or material support centers of critical infrastructure and distributing material flows within its transport and logistics system – whether for evacuating populations from disaster-affected areas or for delivering essential goods from state reserve warehouses to local residents via intermediate collection points or distribution centers. As the mathematical toolkit, the following models and methods are employed: 1) optimal multiplex partitioning of sets; 2) rational selection of service center locations based on existing facilities with multiple coverage of continuously distributed demand; 3) two-stage allocation of resource flows within the transport and logistics system.

The main material of this work is presented according to the following structure: 1) mathematical models and methods for facility location with defined service zones; 2) description of the developed software package implementing the proposed facility location technologies in this context; 3) results of service center location and territorial zoning, particularly for cases involving critical infrastructure facilities.

3. Materials and methods

The mathematical models presented below are constructed under the following assumptions regarding the input data:

1. The logistics centers to be located constitute critical infrastructure in the event of a natural or man-made emergency. They perform functions of material distribution and human resource planning. The construction and maintenance costs of each center are assumed to be known and remain constant throughout the planning horizon.
2. The capacity of each logistics center is limited. Each service center (or intermediate distribution center) is assigned a specific territorial zone. Services may be provided by any of the k nearest centers, ensuring a flexible and reliable logistics chain.
3. The demand for services can be described analytically by a certain function.
4. In the case of two-stage resource allocation, the organizational and setup costs of second-stage centers are assumed to be known for all units. Resources may be human (in the case of evacuation processes) or material (for the delivery of essential goods to destinations). The centers to be located are intermediate facilities (hubs, primary resource collection points, redistribution centers, etc.).
5. The time required for movement between demand points and centers, as well as between centers of the two stages, is directly proportional to the distance. The latter is assumed to be known and invariant over time.
6. All resources accumulated in an intermediate distribution center must be redistributed among the final recipients.

Problem 1. Optimal location of centers with service zone definition (problem of optimal multiplex partitioning of a set with center location and capacity constraints).

Let Ω be a bounded territory; let $\tau_i \in \Omega$, $i = 1,2,\dots,N$, denote certain points referred to as centers (either predefined or to be located); let $\mathrm{N} = \{1,2,\dots,N\}$ be the set of all center indices; let $\mathrm{M}(\mathrm{N},k)$ be the set of all k-element subsets of N, $|\mathrm{M}(\mathrm{N},k)| = C_N^k = L$; let $\rho(x)$ be a nonnegative function describing the demand for service; let $c(x,\tau_i)/w_i$ denote the cost of providing a service at point $x \in \Omega$ by center τ_i (assumed to be proportional to the distance $c(x,\tau_i)$ between the points); let a_i be the cost of establishing a new or upgrading an existing center τ_i, or its fixed organizational costs calculated per unit of demand, $i = \overline{1,N}$; and let $b_1, b_2, \dots, b_N$ be the capacities of the centers, determining the maximum volume of services that the corresponding centers can provide.

The problem is to determine the coordinates of the locations of N centers and to partition the given region into subareas $\Omega_{\sigma_l}, l = \overline{1,L}$, each of which is associated with the same k nearest service centers $\{\tau_{j_1^l}, \tau_{j_2^l}, \dots, \tau_{j_k^l}\}$, in order to minimize the total cost of servicing the entire region. Here $\sigma_l = \{j_1^l, j_2^l, \dots, j_k^l\}$ is the set of indices of centers associated with zone Ω_{σ_l}. The partitioning of Ω must be carried out subject to the capacity of each j-th center and the share γ_j^l of the service market it occupies in territory Ω_{σ_l}, among the set $\{\tau_{j_1^l}, \tau_{j_2^l}, \dots, \tau_{j_k^l}\}$, serving that territory.

Let $\Sigma_\Omega^{N,k}$ denote the class of all possible k-order partitions of Ω into subsets $\Omega_{\sigma_1}, \dots, \Omega_{\sigma_L}$:

$$\Sigma_\Omega^{N,k} = \Big\{\bar{\omega} = \{\Omega_{\sigma_1}, \dots, \Omega_{\sigma_L}\}\colon \bigcup_{l=1}^{L} \Omega_{\sigma_l} = \Omega, \; mes\left(\Omega_{\sigma_i} \cap \Omega_{\sigma_j}\right) = 0, \sigma_l, \sigma_j \in \mathrm{M}(\mathrm{N},k), i \neq j, \; i,j = \overline{1,L}\Big\}. \quad (1)$$

When $k = 1$, this corresponds to a partition of the territory into zones of monopolistic service.

The problem of optimal location of centers with capacity constraints and service zone definition is formalized as follows:

$$F(\bar{\omega}, \boldsymbol{\tau}^N) \to \min_{\bar{\omega} \in \Sigma_\Omega^{N,k};\, \boldsymbol{\tau}^N \in \hat{\Omega}^N}, \quad (2)$$

$$F(\bar{\omega}, \boldsymbol{\tau}^N) = \\ = \frac{1}{k} \sum_{l=1}^{L} \int_{\Omega_{\sigma_l}} \sum_{i \in \sigma_l} (c(x,\tau_i)/w_i + a_i)\rho(x)\, dx, \quad (3)$$

$$\sum_{\substack{l=1 \\ l:\, i \in \sigma_l}}^{L} \int_{\Omega_{\sigma_l}} \gamma_i^l \rho(x) dx = b_i, \quad i = \overline{1,p}; \quad (4)$$

$$\sum_{\substack{l=1 \\ l:\, i \in \sigma_l}}^{L} \int_{\Omega_{\sigma_l}} \gamma_i^l \rho(x) dx \leq b_i, i = \overline{p+1,N}. \quad (5)$$

Here $w_i > 0$, $a_i \geq 0$, $b_i \geq 0$, $i = \overline{1,N}$, are given parameters. The coefficients γ_i^l satisfy the following conditions for all $j = \overline{1,N}$, $l = \overline{1,L}$:

$$0 \leq \gamma_j^l \leq 1, \quad \gamma_{j_1^l}^l + \gamma_{j_2^l}^l + \dots + \gamma_{j_k^l}^l = 1. \quad (6)$$

If the distribution of demand across Ω is proportional to the capacities of the centers, then for all $l = \overline{1,L}$ and for all $j = \overline{1,N}$, $j \in \sigma_l$, the quantities γ_j^l are given by: $\gamma_j^l = b_j / \sum_{q:\, q\in\sigma_l} b_q$. If the demand is distributed evenly among the centers, then $\gamma_j^l = \frac{1}{k}$, for all j and l.

For problem (2) – (6) to be solvable, the total capacity of the centers must cover the service demand, i.e., the following conditions must hold:

$$0 \le b_i \le S, \quad i = \overline{1,N};\ \sum_{i=1}^{p} b_i \le S \le \sum_{i=1}^{N} b_i. \quad (7)$$

A rigorous theoretical justification of the solution method for problem (2) – (6) is provided in Koriashkina et al. (2025). It is shown that, by applying duality theory, one can obtain analytical expressions for the characteristic functions of subsets that form the multiplex partition of Ω. These expressions involve several parameters, which are themselves solutions of an auxiliary nondifferentiable optimization problem. Consequently, the algorithm for solving the formulated problem is based on modern methods of nonsmooth optimization. In the developed software package, Shor's *r*-algorithm is implemented.

If, instead of minimizing (2), we minimize the following functional:

$$F_R(\bar{\omega}, \tau^N) = \max_{l=\overline{1,L}} \sup_{x\in\Omega_{\sigma_l}} \max_{i\in\sigma_l} (c(x,\tau_i)/w_i + a_i)\rho(x) \to \min_{\substack{\bar{\omega}\in\sum_{\Omega}^{N,k},\\ \tau^N\in\Omega^N}}, \quad (8)$$

we obtain the problem of optimal multiple set covering with constraints. For certain input data, (8) defines the coverage radius:

$$R(\tau_*^N) = \inf_{\tau^N} \sup_{x\in\Omega} \min_{i=1,\dots,N} c(x,\tau_i). \quad (9)$$

Methods of multiple spherical covering are developed using elements of the theory of continuous optimal partition problems and likewise reduce to constrained nonsmooth optimization problems. To solve them, subgradient methods and penalty function techniques are used. In addition, "regularization" of the objective functional is applied either to prevent centers from being located too close to one another or to ensure that they are distributed as evenly as possible throughout the territory.

If the capacities of the centers to be located are not known in advance, they can be estimated by solving the multiplex partitioning problem for continuous sets. The capacity of a located center can be determined based on information about the size of the territory assigned to the center and the demand for services within it, considering the multiplicity of partitioning, namely:

$$Capacity(\tau_i) = \sum_{\substack{l=1\\ l:\ i\in\sigma_l}}^{L} \int_{\Omega_{\sigma_l}} \gamma_i^l \rho(x)dx \quad i = \overline{1,N}. \quad (10)$$

where γ_i^l are constants which satisfy conditions (6).

Problem 2. Rational selection of service center locations based on existing facilities (discrete-continuous problem of optimal multiple set covering).

Let $\nu_m \in T,\ m = 1,\dots,M$, $T \subseteq \Omega$, be predefined objects or sites specified by their coordinates and the cost of establishing logistics hubs at them. The number of potential sites exceeds the number of centers to be located. From all possible sites, it is necessary to select *N*, *N* < *M*, in an optimal sense. The budget for establishing new centers is A units, while the organization of a center at point ν_m costs a_m units, $m = \overline{1,M}$.

The problem of optimally selecting centers τ_i from set T and defining their service zones in region Ω with the objective of minimizing the maximum possible service time from any of the *k* nearest centers to a customer, is formally stated as follows: find vector $y* = \{y_m^*\}_{m=\overline{1,M}}$, and the corresponding vector-function $\lambda^*(\cdot)$, such that:

$$\bar{R}\big(y, \lambda(\cdot)\big) = \sup_{x\in\Omega} \min_{\lambda(x)\in\Lambda_M^k} \max_{m=\overline{1,M}} c(x,\nu_m)\lambda_m(x) \to \min, \quad (11)$$

subject to the following constraints:

$$\sum_{m=1}^{M} y_m = N, \quad (12)$$

$$\sum_{m=1}^{M} \lambda_m(x) = k\ \forall x \in \Omega. \quad (13)$$

$$\lambda_m(x) \le y_m\ \forall m = \overline{1,M}, \forall x \in \Omega; \quad (14)$$

$$\lambda_m(x) = 0 \vee 1,\ m = \overline{1,M}; \forall x \in \Omega; \quad (15)$$

$$\sum_{m=1}^{M} a_m y_m \le \mathrm{A}, \quad (16)$$

$$y_m = 0 \vee 1, \quad m = \overline{1,M}, \quad (17)$$

$$\nu_m \in T,\ m = 1,\dots,M. \quad (18)$$

Here, variables y_m and $\lambda_m(x)$ $\forall x \in \Omega$, are binary. Specifically, $y_m = 1$, if a center is located at $v_m \in T$, and $y_m = 0$, if v_m is unused, $m = 1,\dots,M$; while:

$$\lambda_m(x) = \begin{cases} 1, \text{ if } x \text{ is in the service zone of center } v_m, \\ 0 \text{ otherwise}, \ m = \overline{1,M}. \end{cases} \quad (19)$$

In problem (11) – (19), the facilities to be located are assumed identical. If, however, each $\tau_i, i = \overline{1,N}$, and each potential site possesses its own characteristics, then the unknown is a matrix with elements $\beta_{im}, i = \overline{1,N}, m = \overline{1,M}$, such that:

$$\begin{gathered} \beta_{im} = \begin{cases} 1, \text{ if center } \tau_i \text{ is located at site } v_m, \\ 0 \text{ otherwise}, \end{cases} \\ \sum_{i=1}^{N} \beta_{im} \le 1 \ \forall m = \overline{1,M}, \\ \sum_{m=1}^{M} \beta_{im} = 1 \ \forall i = \overline{1,N}. \end{gathered} \quad (20)$$

In this case, the representation is:

$$\tau_i = \sum_{m=1}^{M} \beta_{im} v_m, \ i = \overline{1,N}. \quad (21)$$

A numerical algorithm for solving the formulated problem is presented in Dziuba et al. (2023).

Problem 3. Two-stage distribution of resource flows in a transport and logistics system (optimal partitioning of a continuous set with additional links).

Let Ω be a territory that has suffered (or may suffer) damage as a result of an emergency; $\hat{\Omega} \subseteq \Omega$ is the territory on which intermediate distribution centers may be located; $\rho(x)$ is a function describing the distribution of resources at point $x \in \Omega$, units of resource per unit area; N and M are the numbers of first- and second-stage centers, respectively; S is the total amount of resources in territory Ω, units of resource; τ_i^r is the i-th center of stage r, with the centers of stage I considered intermediate; b_i^r is its capacity (the maximum volume of resources that the respective centers can hold/provide), $r = I, II$, units of resource; $c_i^I(x, \tau_i^I)$ is the cost of delivering resources from point $x \in \Omega$ to center τ_i^I, monetary units per unit of resource; $c_{ij}^{II}(\tau_i^I, \tau_j^{II})$ is the cost of transporting resources from τ_i^I to τ_j^{II}, monetary units per unit of resource; a_i^I are the costs of organizing an intermediate center at point τ_i^I monetary units per unit of resource; a_j^{II} are the fixed organizational costs of center τ_j^{II}, monetary units per unit of resource; v_{ij} is the amount of resource transported from τ_i^I to τ_j^{II} (or in the opposite direction), units of resource, $i = \overline{1,N}$; $j = \overline{1,M}$.

It is required that:

$$F(\bar{\omega}, \tau^I, v) \to \min, \quad (22)$$

$$F(\bar{\omega}, \tau^I, v) = \beta_1 \sum_{i=1}^{N} \int_{\Omega_i} (c_i^I(x, \tau_i) + a_i^I)\, \rho(x) dx + \\ + \beta_2 \sum_{i=1}^{N} \sum_{j=1}^{M} \left(c_{ij}^{II}(\tau_i^I, \tau_j^{II}) + a_j^{II}\right) v_{ij}, \quad (23)$$

$$\int_{\Omega_i} \rho(x) dx = \sum_{j=1}^{M} v_{ij}, \quad i = \overline{1,N}, \quad (24)$$

$$\sum_{i=1}^{N} v_{ij} = b_j^{II}, \quad j = \overline{1,M}, \quad (25)$$

$$\bar{\omega} \in \Sigma_{\Omega}^{N}, \quad v \in R_{NM}^{+}, \quad \tau^I \in \hat{\Omega}^N, \quad (26)$$

$$\Sigma_{\Omega}^{N} = \left\{\bar{\omega} = \{\Omega_1, \Omega_2, \dots, \Omega_N\} : \bigcup_{i=1}^{N} \Omega_i = \Omega, \ mes(\Omega_i \cap \Omega_j) = 0, \ i \ne j, \ i,j = \overline{1,N}\right\}, \quad (27)$$

where $\beta_1, \beta_2 \ge 0$, $\beta_1^2 + \beta_2^2 \ne 0$ are given values that set the priorities of the terms and account for their normalization and dimensionlessness; Σ_{Ω}^{N} is the class of all possible partitions of Ω into N zones; $\bar{\omega} = \{\Omega_1, \Omega_2, \dots, \Omega_N\}$ is an element of Σ_{Ω}^{N}; R_{NM}^{+} is the space of $(N \times M)$ nonnegative real matrices.

The method and numerical algorithm for solving problem (22) – (27) were developed according to the following scheme. Problem (22) – (27) is formulated in an equivalent form with respect to the characteristic functions of the subsets that constitute the partition of Ω. By applying duality theory, these are obtained in analytical form, but their expressions contain several parameters, which are solutions of an auxiliary nonlinear programming problem. This problem is solved using the r-algorithm. In this way, the optimal location of the first-stage centers and the characteristic functions of the subsets forming the optimal partition of Ω are determined. At each step of the r-algorithm: the capacities of the first-stage centers are computed according to the defined service zones; a linear programming problem of the transportation type is solved by the potential method, and the optimal values of the resource flows between the first- and second-stage centers are obtained.

4. Description of the Developed Software Package

The selection of locations for logistics centers should be carried out in a well-grounded manner, using appropriate applied tools. Modern geographic information systems make it possible to create repositories of digital geospatial information for assessing the risks of natural hazards and technological accidents, such as landslides or fires.

Based on the integration of mathematical models and solution methods for location/allocation problems presented above with GIS, a software package has been developed for locating logistics centers with simultaneous determination of their service zones. This GIS-enhanced technology combines geographic data with multiple information layers into interactive maps.

In the developed software application (Fig. 1), the Google Maps API cloud platform is used to obtain spatial data. In particular, the Places, Routes, and Geocoding services are used to obtain points of interest (e.g., schools or hospitals), build routes, and perform reverse geocoding to determine locations by their coordinates.

Based on these data, algorithmic modules are activated to perform multiplex or monopolistic location of resource collection centers. The centers may either be optimally located within the territory or selected from existing POIs. After location, the optimal allocation of resources is computed, and the second-stage transportation problem – resource movement between collection centers – is solved using real routes obtained from Google Maps.

The computations are carried out by a special API request handler that manages calls to algorithms and interaction with the GIS module. All results are stored in a database. After the computations are completed, the results are transferred to the front-end application and displayed to the user. Visualization is performed using OpenStreetMap: collection centers are marked with markers, coverage zones are displayed as polygons, and routes between second-stage centers are shown with arrows. Thus, the user receives a complete picture of territorial location and logistics, taking real geodata into account.

5. Results and Discussion

Figure 2a visualizes the optimal solution of the two-stage distribution problem: 100 resource units, uniformly distributed over a square territory, are collected by N = 15 intermediate centers and delivered to M = 4 final-destination centers with a capacity of 25 units each. Next to each center, the amount of accumulated resource is indicated in parentheses; arrow weights show the magnitudes of the flows directed to the second-stage centers, and dashed lines mark the boundaries of the first-stage service zones. In the optimal solution, several intermediate centers accumulate negligible amounts of resource (less than one unit), so their operation would be irrational in terms of the use of transport vehicles or human resources. To avoid such situations, the developed technology incorporates two sequential procedures for improving the resource distribution scheme.

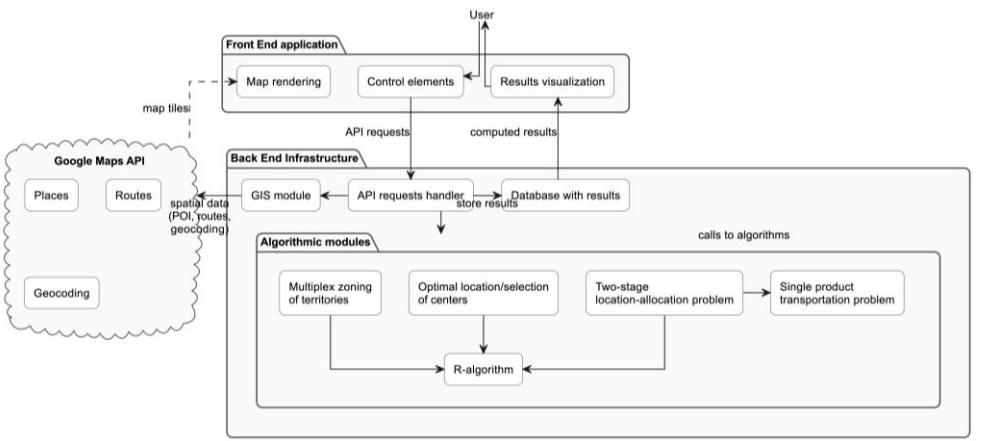


Fig. 1. Software Application Diagram

The first procedure excludes low-capacity intermediate centers from consideration and re-solves the problem, redistributing their resources among the remaining centers. The second procedure prevents small resource flows between the first- and second-stage centers: a lower bound is imposed on the volume of an admissible flow, the capacities of the second-stage centers (calculated by formula (10)) are slightly increased, and the surplus thus introduced is absorbed by a fictitious intermediate center with prohibitively high transportation costs in the extended transportation problem.

The result of applying these rationalization procedures to the optimal solution is shown in Fig. 2b: only 6 of the 15 intermediate centers are retained, while the value of the objective functional increases by less than 1% – from 857.32 to 865.57 (the legends in the figures give the costs of the first and second stages and their sum). This is the main practical conclusion of the two-stage experiments: the structure of the logistics network can be radically simplified – the number of intermediate facilities to be organized, staffed, and maintained is reduced by more than half – at a practically negligible loss of optimality.

Thus, the calculations confirm the expediency of complementing optimal solutions of two-stage location–allocation problems with the proposed rationalization procedures.

We now demonstrate the developed mathematical and software tools, with integrated calls to the geographic information system, on model problems of territorial zoning and optimal location of emergency logistics system units. Figure 3 compares the optimal location of seven centers under monopolistic servicing (k = 1, left) and under duplex coverage (k = 2, right). In the monopolistic case, the optimal service zones coincide with the Voronoi diagram generated by the located centers, in full agreement with the theory of continuous location–allocation problems; for k = 2, the territory is partitioned into second-order Voronoi cells, each of which is assigned to the two nearest centers.

The comparison quantifies the price of coverage reliability: guaranteeing service to every point of the territory by its two nearest centers increases the total cost from 146.82 to 210.61, i.e. by about 43%, while the area of influence of each center expands substantially. The decision-maker thus receives an explicit trade-off between redundancy of coverage and transportation costs – information that classical single-assignment location models do not provide.

The results presented in Fig. 4 confirm the correctness of accounting for the capacity constraints of the centers. Without constraints (left), the monopolistic zones of five fixed centers form a classical Voronoi partition of the territory with a total cost of 156.64. When the capacities of two of the centers are limited (right), their service zones contract to subregions with curvilinear boundaries, the surplus demand is reallocated to the neighboring centers, and the total cost rises to 199.22. Notably, the qualitative change in the geometry of the optimal zones – their boundaries cease to be straight-line segments – is captured analytically by the characteristic functions derived from duality theory, without any heuristic post-processing.

Figures 5 and 6 illustrate the operation of the GIS-integrated software application on a real urban territory. Existing facilities suitable for the role of population collection or goods distribution points – schools, hospitals, kindergartens, and other categories of points of interest – are retrieved automatically from OpenStreetMap for the outlined study area (Fig. 5). For the selected facilities, the multiplex zoning of the territory is then computed: Fig. 6 shows the duplex (k = 2) responsibility zones constructed with the use of real pedestrian routes and population distribution data. Automatic identification of suitable existing objects and of their service areas eliminates the need to involve analysts for a detailed examination of these zones.

It is worth comparing the proposed approach with the location–allocation techniques discussed in the literature review. Discrete formulations – mixed-integer programming models (Li J. et al., 2023; Ljubić et al., 2024) and covering models (Alizadeh et al., 2021) – require demand to be aggregated into a finite set of points and candidate sites to be enumerated in advance, and their computational burden grows rapidly with the numbers of demand points, sites, and hierarchy levels (Clautiaux & Ljubić, 2025). Metaheuristic schemes – genetic algorithms and their modifications (Wang et al., 2021; Serhieiev & Us, 2023; Fu et al., 2024) – scale better but provide no optimality guarantees, while multi-criteria ranking procedures (Stopka et al., 2022) and simulation studies (Szczepański et al., 2019) evaluate a small set of predefined alternatives rather than optimizing over the whole territory.

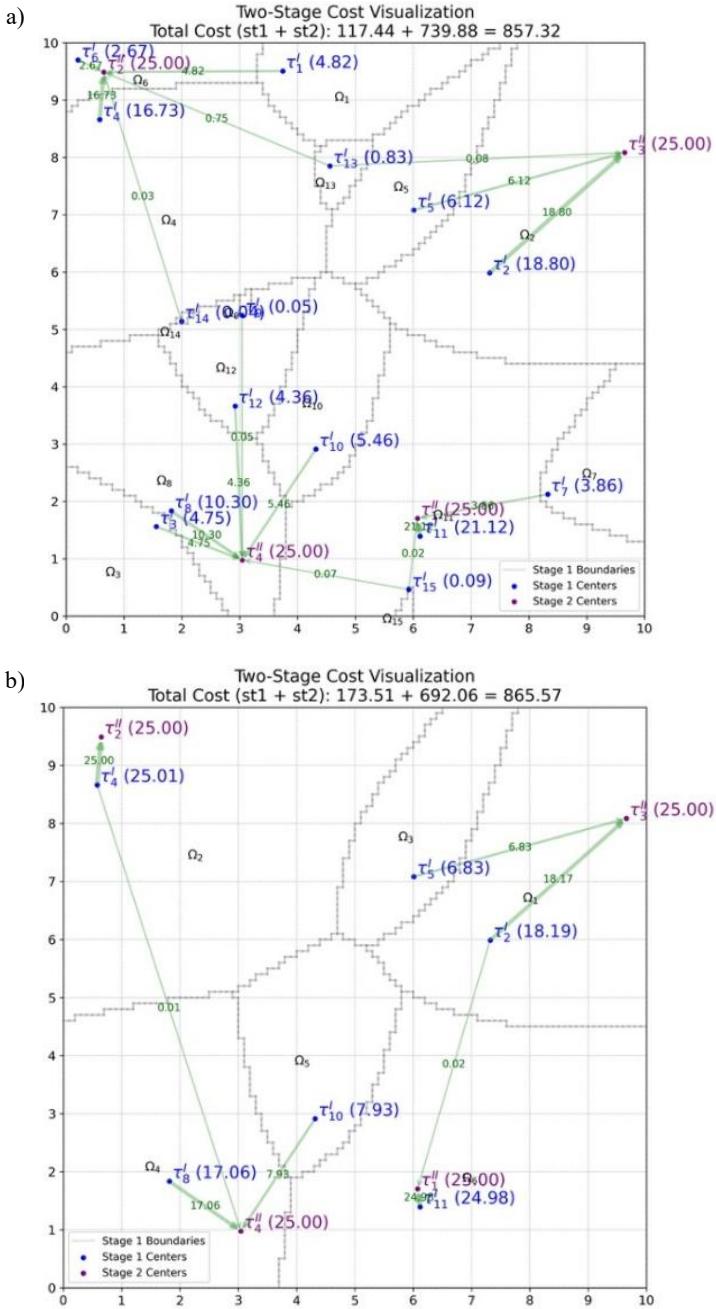
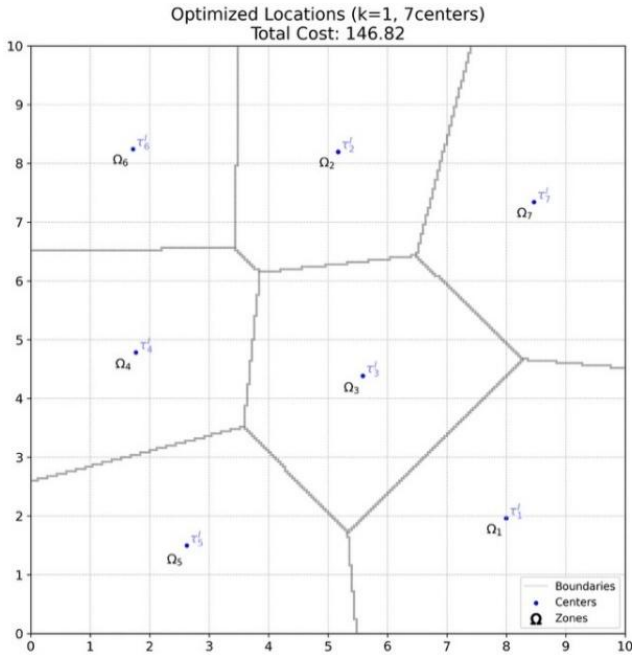


Fig. 2. Two-stage distribution of resources continuously distributed over a square area: a) optimal solution of the problem; b) result of applying the improvement procedure

a)



b)

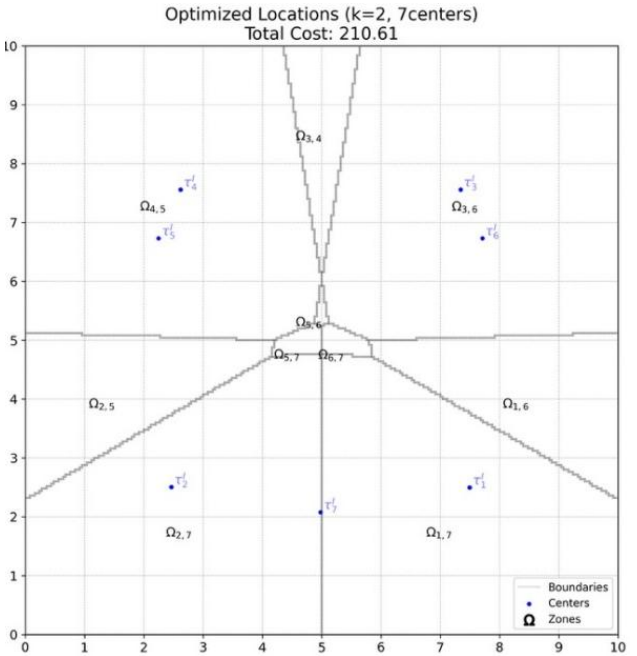
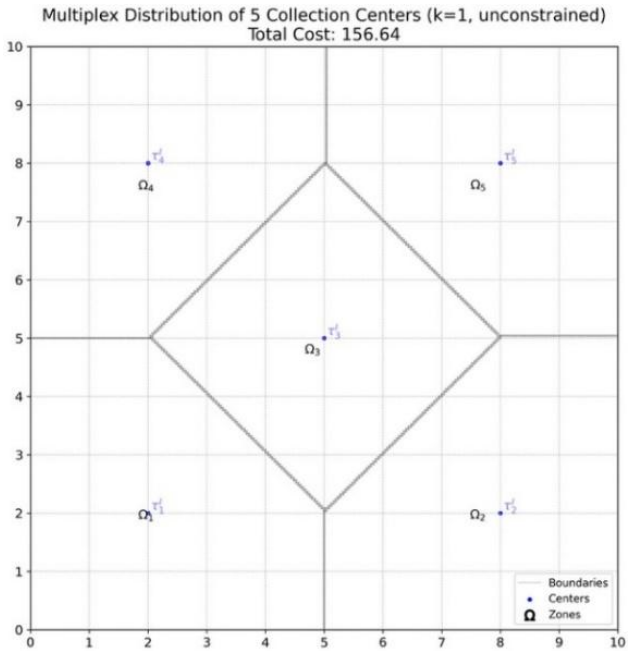


Fig. 3. Optimal location of logistics centers and determination of service zones: a) monopolistic servicing (k = 1); b) duplex coverage (k = 2)

a)



b)

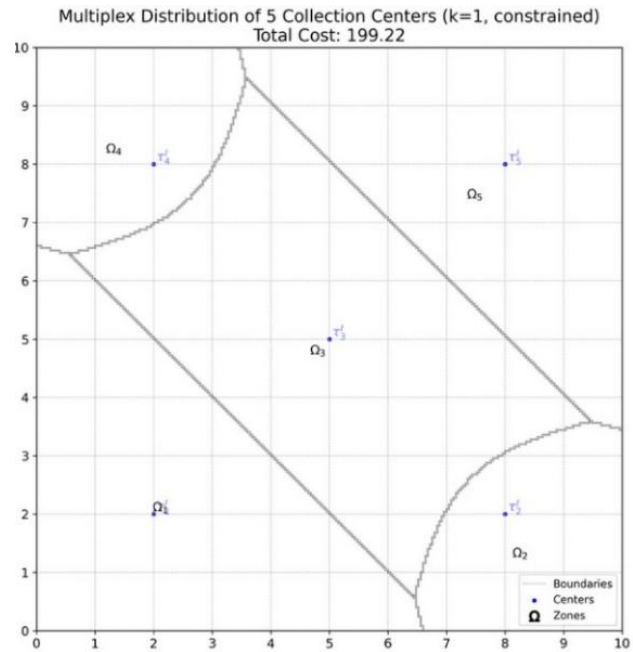


Fig. 4. Accounting for limited capacities of logistics centers: a) zoning without capacity constraints; b) zoning with limited capacities of two centers

In contrast, the technology presented here treats demand as continuously distributed and derives the boundaries of service zones analytically, on the basis of duality theory: the locations of the centers, their service zones, and the interstage flows are obtained simultaneously by nonsmooth optimization (Shor's r-algorithm) without discretizing the territory into demand points and without pre-enumerating candidate zones. A direct numerical comparison with these techniques is complicated by the fact that they solve a discretized problem: its outcome is dominated by the granularity of demand aggregation rather than by the properties of the methods themselves, and the reliability-oriented multiplex formulation with capacity constraints has no established benchmark instances. Systematic quantitative benchmarking on common GIS-based test instances is therefore identified as a separate direction of future work.

From the practical standpoint, the proposed approach offers several advantages over the existing facility-location methods. First, multiplex (k-fold) coverage is embedded in the model itself, so the reliability of service – the assignment of every point of the territory to its k nearest centers – is obtained as a natural outcome of optimization rather than through scenario enumeration. Second, capacity constraints and fixed organizational costs are treated within the same analytical framework, which allowed the rationalization procedures to more than halve the number of intermediate facilities at a cost increase of less than 1%. Third, the direct integration with geoinformation services (POI search, routing, population distribution) turns the mathematical models into an end-to-end decision-support tool: real objects are proposed as candidate centers, and their responsibility zones are drawn on the map automatically, which is particularly valuable for planning proactive measures and for rapid response under emergency time pressure.

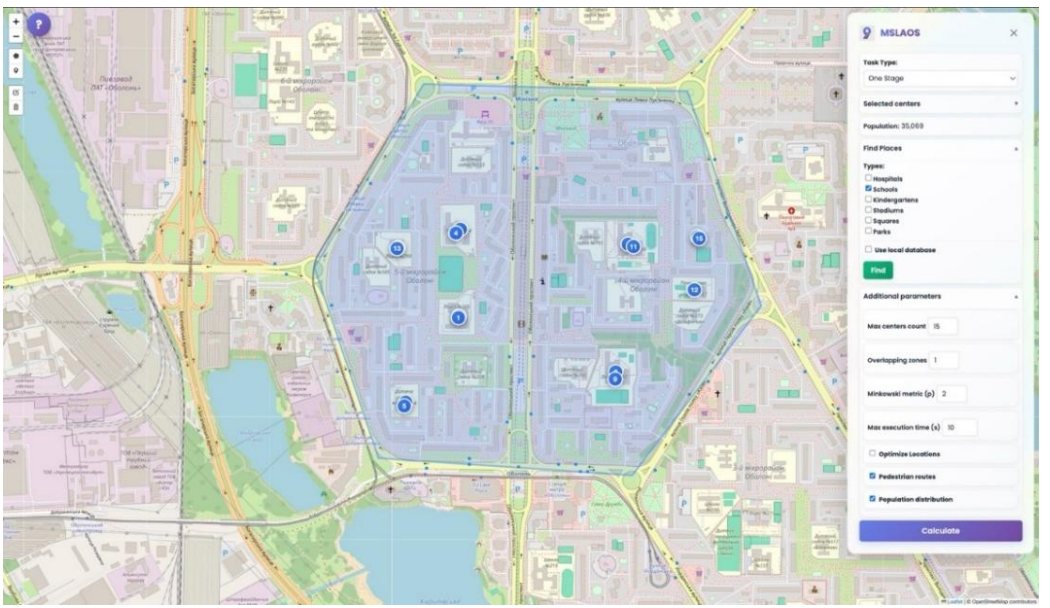


Fig. 5. Automated search for potential logistics centers among existing urban facilities (OpenStreetMap POI) within the study area

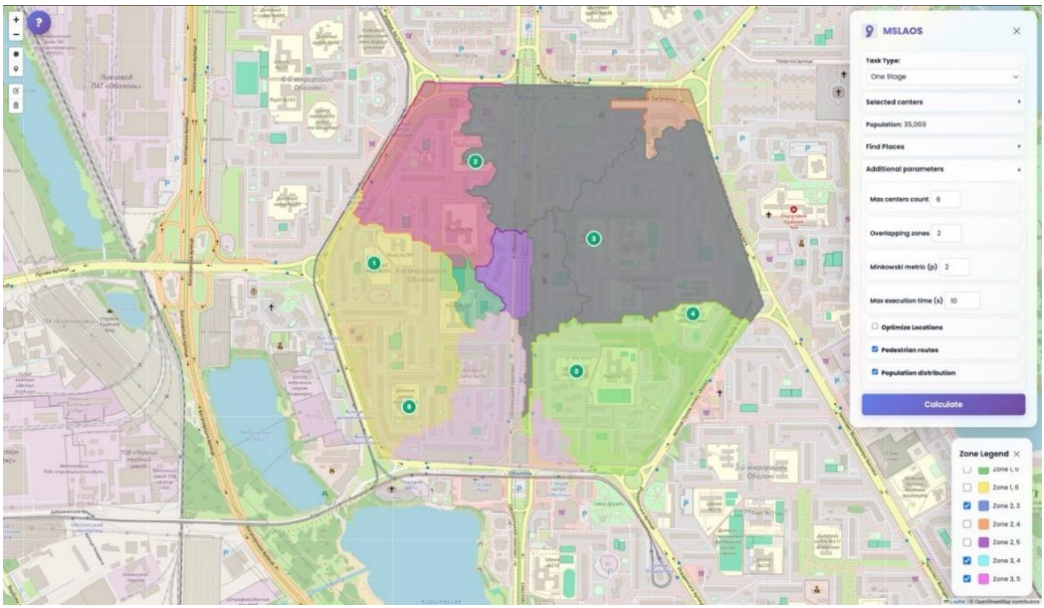


Fig. 6. Multiplex (k = 2) zoning of the territory for logistics centers selected from existing facilities

6. Conclusions

The conducted study confirms the relevance of applying modern mathematical methods and geoinformation technologies to solve problems of optimal location of critical infrastructure facilities and the organization of material flows in emergency situations. A technology has been proposed that is based on models of optimal facility location, multiplex spatial partitioning, and two-stage resource distribution in hierarchical logistics systems. This approach makes it possible to account for center capacity constraints and to minimize total transportation and organizational costs.

The developed technologies and multifunctional software package, integrated with modern GIS, provide the ability to analyze the efficiency of logistics center placement, determine their service zones, and model material resource flows. The integration of Google Maps API services (Places, Routes, Geocoding) for obtaining spatial data ensures automated processing of geolocation information and significantly improves the accuracy and timeliness of decision-making regarding the location of critical infrastructure facilities.

The results of logistics process modeling within the proposed framework confirm its effectiveness and practical value, particularly in supporting managerial decisions aimed at improving the resilience and efficiency of critical infrastructure functioning. The application of the developed models and software enables well-grounded formation of the logistics system structure, optimization of supply routes and service zones, thereby contributing to the overall efficiency of resource management under crisis conditions.

Important directions for future research include the implementation of the developed software package into monitoring, prevention, and emergency response systems, as well as ensuring environmental safety; the improvement of existing and the development of new mathematical models for problems arising in the study of geological processes; the use of GIS technologies for population density assessment; the refinement of interzonal boundaries by aligning them with existing transportation routes; and quantitative benchmarking of the proposed technology against discrete location–allocation models and metaheuristic methods on common GIS-based test instances.

References

1. Alizadeh, R., Nishi, T., Bagherinejad, J., & Bashiri, M. (2021). Multi-Period Maximal Covering Location Problem with Capacitated Facilities and Modules for Natural Disaster Relief Services. *Appl. Sci. 11*, 397. https://doi.org/10.3390/app11010397.
2. Bansal, V., Jayaswal, S., & Sinha, A. (2024). Capacitated multiple allocation hub location problems under the risk of interdiction: model formulations and solution approaches. *Ann Oper Res* 332, 213–251. https://doi.org/10.1007/s10479-023-05563-4.
3. Basallo-Triana, M.J., Vidal-Holguín, C.J., & Bravo-Bastidas, J.J. (2021). Planning and design of intermodal hub networks. A literature review. *Comput. Oper. Res. 136*, 105469. https://doi.org/10.1016/j.cor.2021.105469.
4. Clautiaux, F., & Ljubić, I. (2025). Last fifty years of integer linear programming: A focus on recent practical advances. *European Journal of Operational Research*, 324(3), 707-731. https://doi.org/10.1016/j.ejor.2024.11.018.
5. Chang, K.-H., Hsiung, T.-Y., & Chang, T.-Y. (2022). Multi-Commodity distribution under uncertainty in disaster response phase: Model, solution method, and an empirical study. *European Journal of Operational Research, 303(2)*, 857-876. https://doi.org/10.1016/j.ejor.2022.02.055.
6. Delman, X., Shibeshi, Z., & Scott, M. (2016). Development of a Location Based Service for technician allocation. In *IST-Africa Week Conference* (pp. 1-8). Durban, South Africa. https://doi.org/10.1109/istafrica.2016.7530655.
7. Dziuba, S., Bulat, A., Koriashkina, L., & Blyuss, B. (2023). *Discrete-Continuous Model of the Optimal Location Problem for the Emergency Logistics System*. SSRN: http://dx.doi.org/10.2139/ssrn.4401341.
8. Escudero, L. F., & Monge, J. F. (2021). On Multistage Multiscale Stochastic Capacitated Multiple Allocation Hub Network Expansion Planning. *Mathematics*, *9*(24), 3177. https://doi.org/10.3390/math9243177.
9. Ferraro, S., Cantini, A., Leoni, L., & De Carlo, F. (2023). Sustainable Logistics 4.0: A Study on Selecting the Best Technology for Internal Material Handling. *Sustainability*, *15*(9), 7067. https://doi.org/10.3390/su15097067.
10. Fu, Z., Yue, J., & Yan, Y. (2024). Optimized design of multi-level low-carbon logistics distribution scheme based on two stages. *Archives of Transport*, 69(1), 145-165. https://doi.org/10.61089/aot2024.2mdq5h75.
11. Gao, X., Park, C., Chen, X., Xie, E., Huang, G., & Zhang, D. (2021). Globally Optimal Facility Locations for Continuous-Space Facility Location Problems. *Applied Sciences*, *11*(16), 7321. https://doi.org/10.3390/app11167321.
12. Kiseleva, E.M., Prytomanova, O.M., & Us, S.A. (2020). Solving a two-stage continuous-discrete optimal partitioning-distribution problem with a given position of the subsets centers. *Cybernetics and Systems Analysis*, 56(1), 3-15. https://doi.org/10.1007/s10559-020-00215-y.
13. Koriashkina, L. S., Lubenets, D. E., Minieiev, O. S., & Sazonova, M. S. (2025). Mathematical foundations of methods for solving continuous problems of optimal multiplex partitioning of sets. *Radio Electronics, Computer Science, Control*, (2), 68-82. https://doi.org/10.15588/1607-3274-2025-2-6.
14. Koriashkina, L. S., Dziuba, S. V., Us, S. A., Stanina, O. D., & Odnovol, M. M. (2024). Two-stage problems of optimal location and distribution of the humanitarian logistics system's structural subdivisions. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu,* (1), 130-139. https://doi.org/10.33271/nvngu/2024-1/130.
15. Li, J., Ren, W., & Wang, X. (2023). Joint Location–Allocation Model for Multi-Level Maintenance Service Network in Agriculture. *Applied Sciences*, *13*(18), 10167. https://doi.org/10.3390/app131810167.
16. Li, P., Wen, M., Zu, T., & Kang, R. (2023). A Joint Location–Allocation–Inventory Spare Part Optimization Model for Base-Level Support System with Uncertain Demands. *Axioms*, *12*(1), 46. https://doi.org/10.3390/axioms12010046.

17. Ljubić, I., Pozo, M. A., Puerto, J., & Torrejón, A. (2024). Benders decomposition for the discrete ordered median problem. *European Journal of Operational Research*, 317(3), 858-874. https://doi.org/10.1016/j.ejor.2024.04.030.
18. Lu, F., Yao, L., Wu, K., Hu, Y., & Li, J. (2019). Design of a Reliable Location-Allocation Model under Maintenance Uncertainty for Emergency Service Network. In *IEEE 6th International Conference on Industrial Engineering and Applications (ICIEA)* (pp. 546-550). Tokyo, Japan. https://doi.org/10.1109/iea.2019.8714940.
19. Pirouzi, S., Zadehbagheri, M., Behzadpoor, S. (2024). Optimal placement of distributed generation and distributed automation in the distribution grid based on operation, reliability, and economic objective of distribution system operator. *Electr Eng* 106, 7899-7912. https://doi.org/10.1007/s00202-024-02458-w.
20. Rubio, F., Llopis-Albert, C., & Valero, F. (2021). Multi-objective optimization of costs and energy efficiency associated with autonomous industrial processes for sustainable growth. *Technol. Forecast. Soc. Chang.*, *173*, 121115. https://doi.org/10.1016/j.techfore.2021.121115.
21. Serhieiev, O. S., & Us, S. A. (2023). Modified genetic algorithm approach for solving the two-stage location problem. *Radio Electronics, Computer Science, Control*, 3(63), 159-170. https://doi.org/10.15588/1607-3274-2023-3-16.
22. Stopka, O., Droździel, P., & Ľupták, V. (2022). Modeling the designs in terms of logistics service center placement: a case study. *Archives of Transport*, 64(4), 59-71. https://doi.org/10.5604/01.3001.0016.1049.
23. Szczepański, E., Jachimowski, R., Izdebski, M., & Jacyna-Gołda, I. (2019). Warehouse location problem in supply chain designing: a simulation analysis. *Archives of Transport*, 50(2), 101-110. https://doi.org/10.5604/01.3001.0013.5752.
24. Terentiev, O., Prosyankina-Zharova, T., Diakon, V., & Manuilenco, R. (2023). Development of mathematical models to support decision-making regarding the functioning of critical infrastructure in the industry of energy supply. *Technology Audit and Production Reserves*, 6 (2 (74)), 44-49. https://doi.org/10.15587/2706-5448.2023.293205.
25. Trushkina, N. (2024). Digital strategy of the critical infrastructure development: bibliometric and trend analysis. *International Science Journal of Management, Economics & Finance,* 3(4), 79-100. [in Ukrainian] https://doi.org/10.46299/j.isjmef.20240304.08.
26. Tsopa, V., Cheberyachko, S., Deryugin, O., Litvinova, Y., & Pustovoi, D. (2025). Choosing a Technical Means from the Set of Alternatives by the Method of Gray Relational Analysis (on the Example of a Truck Trailer). In: *Slavinska, O., Danchuk, V., Kunytska, O., Hulchak, O. (eds) Intelligent Transport Systems: Ecology, Safety, Quality, Comfort. ITSESQC 2024. Lecture Notes in Networks and Systems, vol 1336*. Springer, Cham. https://doi.org/10.1007/978-3-031-87379-9_25.
27. Vafaeinejad, A., Bolouri, S., Alesheikh, A. A., Panahi, M., & Lee, C.-W. (2020). The Capacitated Location-Allocation Problem Using the VAOMP (Vector Assignment Ordered Median Problem) Unified Approach in GIS (Geospatial Information Systam). *Applied Sciences*, *10*(23), 8505. https://doi.org/10.3390/app10238505.
28. Wang, Z., Hu, Y., & Ren, W. (2021). A Novel Approach for the Location and Allocation of Maintenance Service Vehicles in Agriculture. In *IEEE 8th International Conference on Industrial Engineering and Applications (ICIEA)* (pp. 37-41). Chengdu, China. https://doi.org/10.1109/iciea52957.2021.9436811.
29. Wu, Sh., Yang, J., Peng, R., & Zhai, Q. (2021). Optimal design of facility allocation and maintenance strategy for a cellular network, *Reliability Engineering & System Safety* 205, 107253. https://doi.org/10.1016/j.ress.2020.107253.
30. Xu, J., Murray, A. T., Church, R. L., & Wei, R. (2023). Service allocation equity in location coverage analytics. *European Journal of Operational Research*, 305(1), 21-37. https://doi.org/10.1016/j.ejor.2022.05.032.
31. Zagurskiy, O. (2021). Modern trends in development of logistics market. *Machinery & Energetics*, 12(1), 17-24. http://dx.doi.org/10.31548/machenergy2021.01.017. Alizadeh, R., Nishi, T., Bagherinejad, J., & Bashiri, M. (2021). Multi-Period Maximal Covering Location Problem with Capacitated Facilities and Modules for Natural Disaster Relief Services. *Appl. Sci. 11*, 397. https://doi.org/10.3390/app11010397.