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ASSESSMENT OF A CRITICAL AREA FOR A GIVE-WAY SHIP IN A COLLISION ENCOUNTER

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Abstract: *Evasive action in ship-ship encounter has to be carried out on time and in line with the international convention on collision regulation – COLREG. The convention not only includes a set of rules defining types of relations between encountering ships but also indicates appropriate action to be taken in a given encounter. One of such encounters is crossing, where, in case of a collision situation, a give-way ship has to take an appropriate action in due time. However, a stand-on vessel is also given an opportunity to manoeuver, if it is made clear to her that the other ship is not fulfilling her obligations. However, it is difficult to specify, at which point in time in the course of an encounter, the stand-on ship has to take an action in order to avoid collision. It is understandable, as this parameter depends on numerous factors, both endogenous (e.g. ship characteristics, her maneuverability), and exogenous (e.g. type of encounter, weather conditions).*
Therefore in this paper we make an attempt towards the definition of the critical area for a maneuver of a stand-on ship, in the situation where the give-way vessel does not take an action. This is determined with the use of a hydrodynamic model of ship motion, and series of simulations conducted for several types of encountering ships under various conditions.
Once determined, the critical area demarcates the no-go area around the own ship, where any other ships on collision courses must not enter. Otherwise a collision cannot be avoided by an action of one ship alone.

Key words: *collision risk, maritime traffic safety, anti-collision, ship motion model, ship domain*

1. Introduction

Ship collisions pose risk to marine environment and human life, especially in busy waterways and sensitive sea areas. Various countermeasures exist to support collision prevention, including training tools (Chauvin, Clostermann, and Hoc, 2009), technology for maritime surveillance (Bukhari et al., 2013) and for integrated navigation support services (Hanninen et al., 2014).

From the operational viewpoint a number of collision avoidance system (CAS) methods have been proposed, in line with developments in e-Navigation (Patraiko, Wake, and Weintrit, 2010). However the most widely used CAS is the Automatic Radar Plotting Aid (ARPA). This technology tracks several targets and displays proximity indicators, called CPA and TCPA, used for operational risk assessment.

Another type of proximity indicator, initially used for the strategic safety assessment, is rooted in a concept of ship domain. Even though it was not developed for the purpose of operational risk assessment (Fujii and Tanaka, 1971; Goodwin, 1973), it finds its application in this field as well, for the recent developments see for example (Kao et al., 2007; Pietrzykowski and Uriasz, 2009; Wang, 2010).

However, the above mentioned proximity indicators are rather subjective, and refer to the comfort area rather than critical area for a ship. The difference between these two areas is substantial, and a navigator handling a ship should be aware of the dimension of the latter, since it would be helpful when planning an evasive maneuver in an encounter, where the other, give-a-way vessel is not acting as supposed. This critical area depends on numerous factors, where a ship dynamics is one of them. Interestingly, only a few studies take into account ship dynamics, as a factor determining the safe area for a given type of a manoeuver, see for example (Colley, Curtis, and Stockel, 1983; Curtis, 1986; Montewka, Goerlandt, and Kujala, 2012; Zhang et al., 2012; Łukaszewicz, 2007). The model by (Colley, Curtis, and Stockel, 1983) uses concept of maneuvering time, domains and arenas to determine analytically the safe distance for the last chance manoeuver. However ship dynamics is implicitly considered in very simplified manner, by calculating the time required for evasive action assuming a fixed rate of turn at a fixed rudder angle of 20 degrees and reaction time of a helmsman. The model by (Curtis, 1986) considers ship dynamics in order to determinate the safe distance for overtaking, however the focus in on a very large crude carrier (VLCC). The model proposed by (Zhang et al., 2012) estimates minimum required distance for collision evasive action evaluated for one scenario only adopting a simplified model of ship dynamics. In our earlier work (Montewka, Goerlandt, and Kujala, 2012), we proposed a model that accounts for several ship types and their dynamics as well as wide number of encounter scenarios, however its scope is different from the former models, since it estimates the probability of collision between ships in the high seas. In the work of (Łukaszewicz, 2007) the critical distance for a passenger ship is determined, however only a few collision encounters were analyzed.

In this paper, we propose an approach to the evaluation of the critical area, which is defined by a set of critical distances between two ships in a collision encounter. The critical area is defined for a collision encounter, where a give-way vessel is not fulfilling her obligations, and a stand-on ship performs the collision evasive action, by substantial course alteration. This approach accounts for ship dynamics in a wide range of crossing encounters. For this purpose we carried out experiments with the use of three dimensional ship motion model evaluated for over 2000 ship-ship encounter scenarios and three types of ship (RoPax, containers carrier and passengers ship).

The results are presented graphically, and the critical area around a stand-on ships, which shall be kept clear from any other ship when on collision courses. The concept presented here could be used in practice, to inform the navigator about available time to reach the critical area in ship-ship collision situation, rather than zero distance, as it is commonly adopted in practice nowadays. It could be implemented as a part of collision evasive solutions or a part of e-navigation systems, raising the awareness of a bridge team.

The paper is structured as follows: Section 1 presents the concept and implemented algorithms to operationalize it. The case study and the obtained results are presented in Section 2. The comparison of the results with some other related concept is performed in Section 3, while Section 4 discusses and concludes the paper.

2. A concept of the critical area for a ship in crossing encounter

The critical moment in any ship-ship encounter is when the development of an encounter becomes dangerous and the stand-on ship has to take an action in order to evade an imminent collision, as prescribed by COLREG, (IMO, 2003).

The negligence related to this decision may lead to close- quarters situation or a collision like for instance in case of car carrier Baltic Ace colliding with the containers vessel Corvus J on 5-th of December 2012, causing the loss of the former with her eleven crewmembers. A give-way ship m/s Corvus J did not fulfill her duties towards a stand-on vessel m/s Baltic Ace as prescribed by collision regulations – COLREG - and continued with her course and speed. Since she failed with her obligation, m/s Baltic Ace, even though she was a stand-on ship in this encounter, had to initiate collision evasive action. Unfortunately, her action was taken too late, (“Baltic Ace Death Toll - Maritime Bulletin” 2012). This situation shows a suspicion of encroachment of the critical area of m/s Baltic Ace, for a successful evasive manoeuver in this particular encounter.

A critical area around a stand-on ship is defined by a set of critical distances along a set of relative bearing from a stand-on ship outwards. A critical distance between two encountering ships is defined here as the shortest distance between two ships on collision courses at which effective evasive last-minute actions can be successfully taken by one vessel alone. If the distance between these ships becomes less than the critical distance, the collision can't be avoided by an action of stand-on ship alone. The critical distance depends on numerous factors, however the most important are – see Figure 1: the angle of courses intersection in an encounter (a), the relative bearing from a stand-on ship to another (b), the maneuverability of a ship which is related to the ship type, size and design (c). A combination of all these factors yields a specific encounter scenario for two ships. For each encounter scenario a model including ship dynamics is executed, and one critical distance is determined. The model is repeated for a large number of scenarios, and a generalized, statistically significant model of critical distance is obtained. Finally, for each relative bearing considered the critical distance is calculated at the 95% confidence interval, forming a critical area in front of a ship. In this study, we estimate the critical area for a containers carrier encountering three other ship types proceeding with various speeds. While simulating the collision encounters, the container carrier is a stand-on ship, the others are considered give-way ships not acting as supposed.

In the following subsections theoretical foundations for the development of critical distance are presented, rooted in COLREG. Also an algorithm evaluating the critical distance and the critical area is shown.

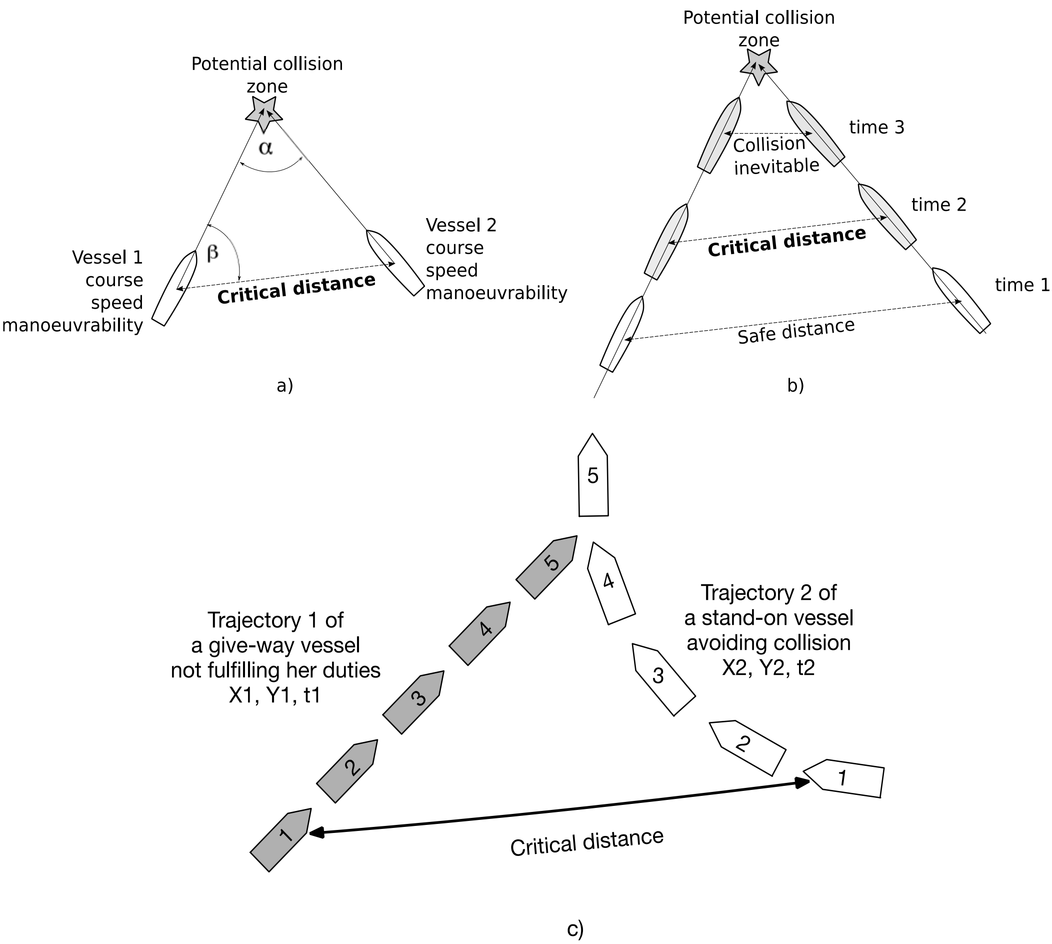

Fig. 1. A graphical representation of a concept of critical distance in ship-ship collision encounter.

3. A mathematical formulation of the concept

A mathematical formulation of the concept analyzed here is based on the tempo-spatial relation between encountering ships' trajectories. The trajectory of each vessel is computed with the use of a state-of-the-art hydrodynamic model, see (Matusiak, 2011). The trajectories of both vessels are described by the following formula (1), discretized in time:

$$\begin{gathered} Trajectory1:\left(X_{Vessel1},Y_{Vessel1},t_n\right) \\ Trajectory2:\left(X_{Vessel2},Y_{Vessel2},t_n\right) \\ t_i=\left\{t_0,t_1,\ldots,t_n\right\} \end{gathered} \tag{1}$$

where: X, Y –vessels' position co-ordinates in a local two-dimensional reference system, in meters; t_n – consecutive time steps in seconds, from the initial moment of the maneuver t_0 to the last moment t_i reflecting passing by of both ships of their collision. Subsequently, these trajectories are inputted to an algorithm determining the critical distance, as presented in Figure 4. The critical distance (*CD*) needs to be determined, for selected ships encountering at the collision courses, according to the formula:

$$\begin{gathered} CD\left(t_{i-j}\right)= \\ d\left(Trajectory1\left(t_{i-j}\right),Trajectory2\left(t_{i-j}\right)\right)\Leftrightarrow \\ \exists!\left(i\in\left(0,n\right)\right)\left(Trajectory1\left(t_i\right)=Trajectory2\left(t_i\right)\right), \\ i,j\in\left\{0,1,\ldots,n\right\} \end{gathered} \tag{2}$$

where d denotes the distance in meters, between trajectories of two encountering ships.
The critical distance is determined with the use of an applied reverse iterative algorithm, which is described in the following section.

4. An algorithm evaluating the critical distance

The objective of this study is to find the critical distance between two ships being on collision courses, at which the collision situation can be solved by one ship performing course alteration. In order to calculate the critical distance for a given encounter scenario, an iterative algorithm is used, as depicted in Figure 4. The basic assumption is that the two ships collide at a time instant ti. The aim of the algorithm is to find such an initial distance between two encountering ships steaming on collision courses, that if a give-way vessel is not reacting, meaning she keeps her speed and course, and the stand-on ship applies rudder angle 20 degrees, which will be kept in the course of the encounter, the ships will be able to avoid collision.
The algorithm initiates its calculations from an instant where two ships already collide, at time ti - their corresponding trajectories have at least one point in common. From this time, the reverse iterative algorithm is applied, which uses a backward calculation method in the space domain. At each iteration, the ships' contours are plotted every second along their predefined trajectories. If two corresponding contours along the trajectories have at least one common point, indicating that they collided, the algorithm increases the initial distance between these two ships at time instant t0 by a constant value, and the trajectories are redrawn. They are starting from the new initial positions of the ships, which are obtained by moving ship B away from ship A along the line of a given relative bearing. For the simplicity of calculations it is assumed that one of the ships holds her initial position (0,0), while the other ship is displaced along the line. The iterations are performed until the contours of the two ships have no overlaps at any time instant for a given relative bearing. In such a situation, the initial positions at time t0 of vessels are recorded and the distance between the ships is calculated and stored. This distance is called critical distance for a given relative bearing. For each relative bearing (labelled as β in Figure 1), 17 crossing angles are considered (labelled as α in Figure 1). This results in 17 values of crossing distances for one β. To calculate the critical distance for a given β, the maximum value of the 17 records is taken.
Then the procedure is repeated for all eight relative bearings, yielding eight values of the critical distances for one side of a ship. The values for another side are considered symmetric. By joining all the points representing the critical distances a critical area is obtained for a given ship-ship encounter. In the case study presented here, we analyzed 9 types of encounters attributed by a type of encountering ships and their speed. The own ship is a small container carrier (LOA=150 m), proceeding with two speed settings: 20 kn and 17 kn, encountering three other ship types (RoRo, container carrier and passengers ship). The particulars of the ships are listed in Table 1.

Table 1. The main particulars of the analysed ships

Ship type	LOA [m]	B [m]	T [m]	V [kn]
Container carrier	150	27.2	8.5	20; 17
RoPax	158	25.0	6.1	20; 18
Passenger ship	185	27.7	6.5	25

Table 2. The analysed encounter scenarios

No	Own ship	Target ship
1	Container carrier 20 kn	Container carrier 20 kn
2	Container carrier 20 kn	RoRo 18 kn
3	Container carrier 20 kn	Passenger 25 kn
4	Container carrier 20 kn	Container carrier 17 kn
5	Container carrier 20 kn	RoRo 20 kn
6	Container carrier 17 kn	RoRo 20 kn
7	Container carrier 17 kn	Container carrier 17 kn
8	Container carrier 17 kn	Pass 25 kn
9	Container carrier 17 kn	RoRo 18 kn
10	Container carrier 17 kn	Container carrier 20 kn

The following assumptions are made here, while evaluating the critical distance:

– the influence of weather conditions is omitted;
– the ships are fully laden;
– the settings of the ships' engines and rudders are constant during the maneuvers. This means that the own ship evades a collision by applying certain rudder angle, which is constant and does not change in the course of collision evasive action;
– the stand-on ship is turning away from the other vessel, which shortens the time at close quarters.

The latter assumption may not always hold in case of timely taken evasive action, however it seems reasonable and acceptable in a case of so called “last chance maneuver”, which is described in the rule 16 and 17 of COLREG. This type of manoeuver is performed with one objective: to avoid a collision by all means. Therefore the action taken requires substantial course alteration in relatively short time.

5. A mathematical model of ship dynamics

The paths of two ships during a collision evasive manoeuver are obtained with the use of a quasi-linear modular hydrodynamic model, see (Matusiak, 2007). For the sake of computational efficiency and the requited level of granularity of the model output, the hydrodynamic model of ship motions is reduced to planar motion neglecting all vertical motions, as well as the shallow water effects. Ship motion is calculated from the following generalized formulae:

$$\begin{cases} \dfrac{du}{dt} = \dfrac{(X_{resist} + X_{prop} + mrv)}{(m - X_{\dot{u}})} \\ \dfrac{dv}{dt} = \dfrac{(-mru + Y_v v + Y_r r + Y_{\dot{r}} \dot{r} + L)}{(m\text{-}Y_{\dot{v}})} \\ \dfrac{dr}{dt} = \dfrac{(N_v v + N_r r + N_{\dot{v}} \dot{v} + LL_{PP} / 0.5)}{(I_{zz}\text{-}N_{\dot{r}})} \end{cases} \tag{3}$$

where:
u - is the linear velocity along axis X; X_{resist} - means the ship resistance; X_{prop} - is the propeller thrust; m - denotes the ship mass; r - is the angular velocity along axis Z; v - is the linear velocity along axis Y; L - means the hydrodynamic lift forces on the rudder; L_{pp} - is the length of the ship between perpendiculars; I_{zz} - is a mass moment of inertia; $X_{\dot{u}}$, Y_v , Y_r , $Y_{\dot{r}}$, $Y_{\dot{v}}$, N_v , N_r , $N_{\dot{v}}$, $N_{\dot{r}}$ are hydrodynamics derivatives and are calculated using formulae derived from the literature, (Artyszuk, 2000; Brix, 1993; Molland and Turnock, 2007).

Three major ship types are considered here, as presented in Table 1, and two values of speed are used to calculate ship trajectories. These are obtained from the historical AIS data recorded in the Northern Baltic Sea, where one value corresponds to the mean value of a distribution for a given ship type and another value corresponds to the 95th percentile of this distribution.

Some of the detailed, ship-specific data, which were required as an input to the ship motion model, were extracted from ships databases. These were draught, service speed, displacement, and main engine power. Two databases were studied, one provided by the Japanese ship classification society Nippon Kaiji Kyokai, from which data concerning container carriers, and ro-ro vessels were extracted. Another source of particulars on vessels was the bulletins issued by the operators of passenger vessels, which cruise in the Northern Baltic Sea. For the detailed description of the adopted ship motion model and its parameters the reader is referred to the earlier work of ours, (Montewka et al., 2010).

Finally, the mathematical model of ship motion is used to determine the trajectories of two encountering ships. These are recorded as files and used as inputs for an algorithm determining the critical distance, as presented in Figure 2.

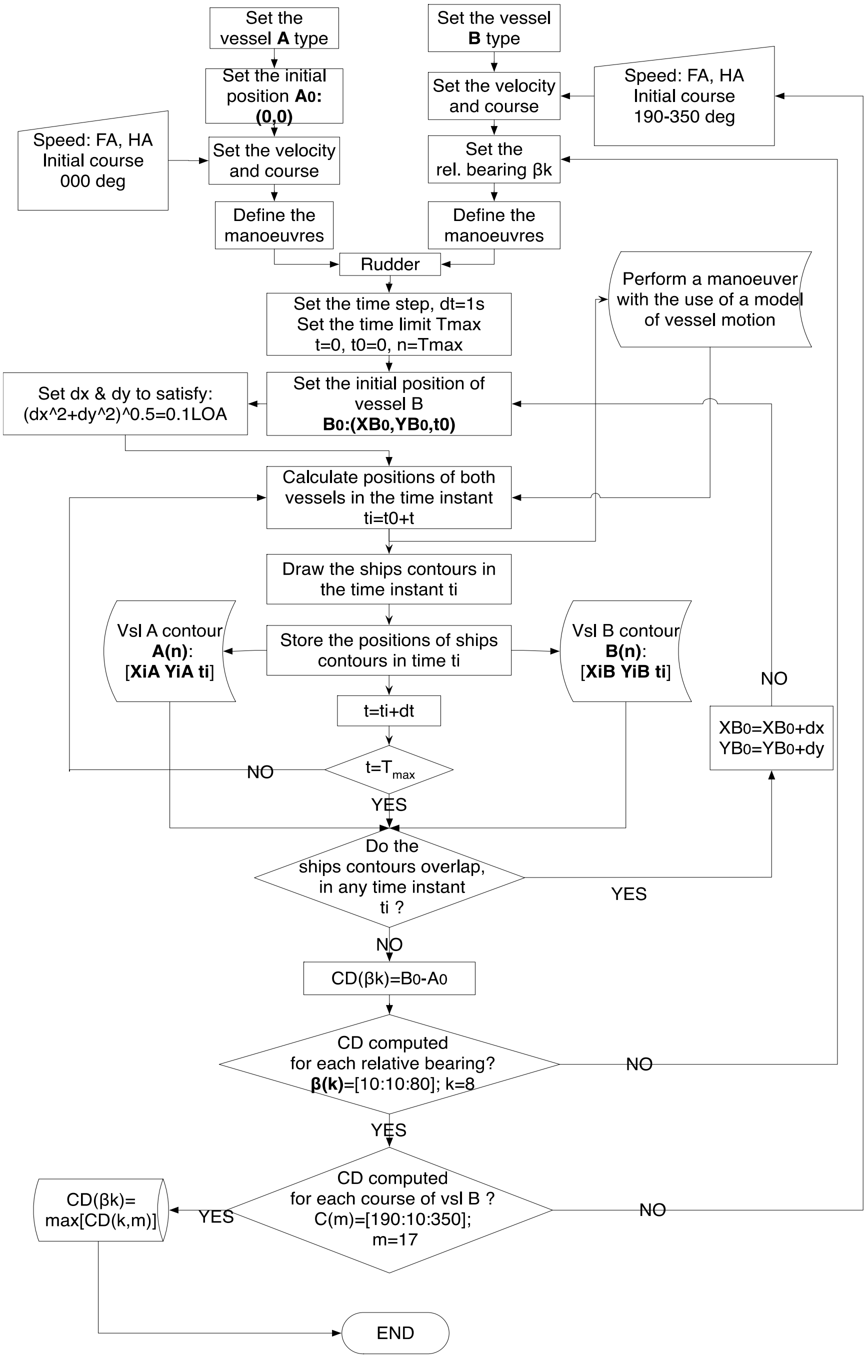

Figure 2. An algorithm adopted to evaluate critical distance.

6. Probabilistic critical area

The algorithm presented in Figure 4 allows evaluation of a critical area for a single encounter. The critical area corresponds to a set of deterministically calculated critical distance. However, in the course of simulations a set of encounters is analysed and set of critical areas is obtained. Based on this data we perform statistical analysis resulting in development of a probabilistic critical area, delimited by 95% confidence interval.

Since the number of analyzed encounters is relatively small, the obtained set of critical areas is scattered and none of commonly known distributions, neither continuous nor discreet, fit the data set. In order to quantify the potential dispersion of an average value and standard deviation for the data set, a non-parametric bootstrap procedure is performed.

In order to develop a probabilistic critical area the mean and standard deviation of a critical distance for a given relative bearing need to be estimated, following the procedure:

– for each relative bearing collect the data set of *n* samples, where *n* is a number analyzed encounters, where one encounter produces one values of critical distances (CD) for a given relative bearing *{CD_1...CD_n}*;
– create *B* bootstrap samples {*CD*_1...CD*_n*}, where each *CD*_i* is a random sample with replacement from {*CD_1...CD_n*}, in our case *B=10^6*;
– estimate, for each bootstrap sample {*CD*_1...CD*_n*}, the required statistics which represents the bootstrap estimate about the expected value of *CD* (μ*) with the associated uncertainty about the expected value (ε*) and standard deviation (σ*);
– based on the expected value of *CD*, uncertainty band and an average standard deviation existing in the sample, the 95% confidence level is determined.

The probabilistic CD for a single relative bearing is determined as follows:

$$\mathrm{CD} = \left(\mu^* + \varepsilon^*\right) + 1.96\sigma^* \tag{4}$$

The probabilistic critical area (CA) is described as a set of CDs for all analyzed relative bearings:

$$\mathbf{CA} = \mathrm{CD}(\beta) \tag{5}$$

The probabilistic critical area developed for the case studies analysed in this paper is presented in Figure 3. The ship is represented as an ellipse in the middle of the graph with an arrow pointing out her heading. The CD is non-dimensional, and it is expressed in ship length, referring to the own ship. The depicted critical area delineates the area that shall be kept clear in collision-situations. This means that the collision evasive actions shall be taken when two encountering ships are beyond the critical envelope.

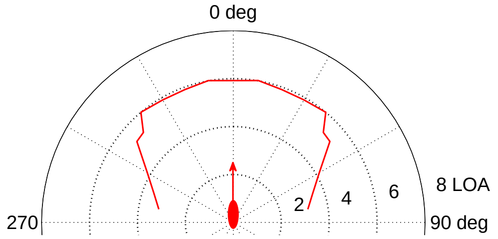

Fig. 3. An exemplary critical area, obtained for a case study presented here.

7. Validation of the results

A full validation process of a model involving element of human, consists of several levels, see for example (Drost 2011). However for the purpose of this study, we demonstrate only one aspect of the validation process, namely concurrent validity. This aspect refers to a comparison of the obtained results with the results of the available studies, which are deemed compatible. For this purpose, we compare the dimension of critical area with dimensions of various ship domains, as found in the literature. The ship domain refers to an area around own ship that an officer wants to keep clear of the other vessels, see for example (Wang et al., 2009; Pietrzykowski and Uriasz, 2009; Hansen et al., 2013).

Some authors claim, that any violation of the ship domain can be interpreted as a threat to navigational safety, (Pietrzykowski and Uriasz, 2009). Thus a concept of ship domain can be considered compatible (to some extent) with the critical area concept presented here.

The ship domain is usually determined in two dimensions and is expressed either in nautical miles or is non-dimensional and the reference is to the own ship length is made. Four ship domains were selected for comparison, as follows:

– A well known, widely used and most probably the oldest domain proposed by (Fujii and Tanaka, 1971), based on empirical studies of maritime traffic around Japan, defined as an ellipse with the following major and minor semi-axes (4.0, 1.6 LOA).

– A domain proposed by (Coldwell, 1983), defined as an ellipse with the following major and minor semi-axes (6.0, 1.75 LOA).
– An empirical domain proposed by (Hansen et al., 2013), which is based on extensive analysis of maritime traffic in the Danish Straits. The domain is defined as an elliptical shape, extending 4.5 LOA in front of a ship, 1.7 LOA aside and 3.5 LOA astern. The domain in this study is referred to *a comfort zone*.
– A fuzzy domain by (Pietrzykowski and Uriasz, 2009), based on a large number of scenarios simulated in a full mission ship bridge simulator, yielding an area around a ship with the following dimensions: 9 LOA in front, 5 LOA to the side and 3 LOA to the stern.

The results of comparison are depicted in Figure 4. The dimensions of the critical area seem to be somewhat larger than the other ship domains. Since the ship domains take usually an elliptical shape, they are slimmer than the critical envelope presented here. This may raise concern about their usability for collision avoidance purpose since they will classify the dangerous cases as safe. Only one of the compared domains by (Pietrzykowski and Uriasz, 2009) is bigger than the critical envelope. It accounts for the human perception of the encounter rather than ship dynamics, and its primary use is to support regular collision avoidance in open sea areas.

8. Discussion and conclusions

In this paper, we propose a concept of critical area for a ship involved in a collision encounter, where a stand-on ship takes the collision evasive action, since the give-way vessel does not fulfill her obligations. The case study presented involves a small container carrier (the stand on ship), three types of give-way vessels and a range of speed at which these ships are proceeding.

The critical area is determined with the use of three dimensional ship motion model evaluated for over 2000 ship-ship encounter scenarios. Based on this results a probabilistic, critical area around the stand-on ship is obtained, which delineates the space required to perform a collision evasive maneuver by this ship alone.

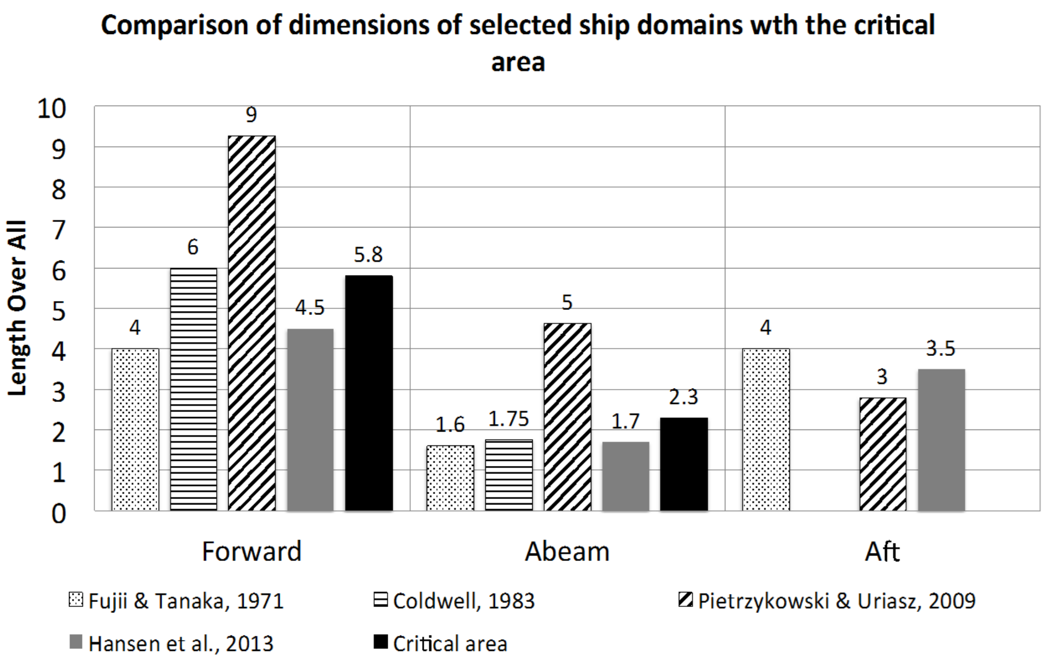

Fig. 4. Comparison of the critical area and selected ship domains.

The dimensions of the critical area, which is obtained, are further compared with the dimensions of ship domains. Since the ship domain is an area around a ship, which navigator wants to keep clear in order to maintain the safe navigation of the own ship, its dimensions shall be wide enough to enable effective evasive manoeuver when a collision is imminent. However, the results of the comparison made here may raise concern about the validity of this statement. This holds especially for the elliptical ship domains. Three out of four analyzed elliptical ship domains are significantly slimmer in the bow-shoulder sectors of a ship than the critical area presented here. This means, that if the two ships are on collision courses, and they are within a distance as defined by a ship domain, in certain sectors they may not be able to avoid collision. Therefore, the application of ship domains to operational risk assessment is rather questionable.

The presented concept shall be further applied to other ship types (different maneuvering characteristics) and other types of encounter namely head-on and overtaking. Also the effect of various strategies of performing collision evasive action on the dimension of the critical envelope is worth studying. The effect of higher variation in speed of encountering ships would be worth an analysis. Moreover, the quantification of the effect of assumptions with respect to hydrodynamic coefficients adopted in the ship dynamic model on the obtained critical areas is of interest.

The solutions stemming from the wider analysis of the proposed concept could be implemented on board ships, as a simple yet useful tool increasing the awareness of navigators with respect to the required maneuvering space to perform evasive action. Ultimately, this may contribute to the improvement of the safety of sea navigation.

Finally, the concept of critical area could be used as a collision criterion in operational and strategic risk assessment models.

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