THE ARCHIVES OF TRANSPORT

VOL. XXIV NO 4 2012

10.2478/v10174-012-0034-0

Maintenance Strategy Maximising Availability Rate

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Received October 2012

Abstract

Electronic equipment operates under various conditions. Due to characteristic nature of its applications (e.g. in transport), it should be highly reliable. Many years' worth of observations show, those systems not only require their constituting parts to function up to par, but also their maintenance has to be efficiently managed. This paper presents maintenance strategies and particularly focuses on maintenance strategy enabling maximising the availability rate.

1. Introduction

The issue of maintaining electronic equipment particularly that used in transport systems is an important problem. This stems from the fact correct reliability and operating parameters have to be assured. Many renowned papers have already been written on the matter [1,2,3,4,5,22,23,24,25,26,27]. By carrying out an adequate reliability analysis of systems, their reliability structures are determined which provide correct reliability parameters. This applies both to the entire system [7,8,9,10,17], as well as its constitution elements e.g. power supply [6,20] and transmission media [19]). Due to this approach, the designed system becomes more reliable. It does not, however, assure high enough availability of the system. Hence, maintenance analysis has to be carried out taking account of selected operating properties of the systems (e.g. failure rate, routine maintenance intensity). Findings of that analysis should enable to fine-tune the maintenance strategy, including rationalisation of routine inspections and their length relative to requirements to those systems in respect of

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their availability in the transport process [12,15,18,21]. The costs it generates are also factored in by the strategy [13,14,16].

2. Maintenance Strategies

Maintenance strategy is the underlying operating procedure for the system which normally comes from research. Its goal is to reach a desired state in the maintenance system. The maintenance process can run uninterruptedly without complete information on current system state and operating conditions.

When systems operate continuously without downtimes, events occur which affect components and equipment forming said system.

2.1. Resource based strategy

Fundamental assumptions to this strategy are:

- predetermined scope of maintenance activities assigned to particular maintenance process,
- maintenance on a regular basis,
- prioritisation of maintenance and repairs.

Dates and remit of maintenance remain constant across the entire strategy. They are usually determined based on years of field tests. They are also independent of technical condition of equipment. Figure 1 presents an example of maintenance cycle.

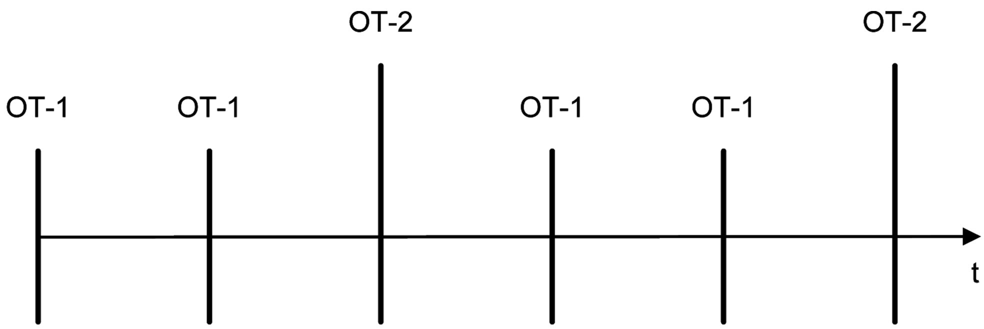

Fig. 1. Example of maintenance cycle
Denotations in figures: OT-1 – on-going maintenance, OT-2 – average maintenance

Prioritisation of maintenance and other activities means that higher ranked maintenance contains lower ranked maintenance activities, e.g. OT-2 contains OT-1 activities.

Technical condition of equipment at time t,

$$t \in [t_0, t_k]$$

could be given by the equation

$$S(t) = f\,[t_0, t, S(t_0), u(t)]$$

where:

S(t) – technical condition of equipment at time t,

$S(t_0)$ – technical condition of equipment at time t_0,

u(t) – overrides in the system during time period $(t\text{-}t_0)$.

Hence, technical condition of a system at any given time t is the function of its initial state at time t_0 and impact of overriding factors during time period $(t\text{-}t_0)$.

Technical conditions of different systems at time t can vary substantially. This stems from the fact each system at time t_0 was in different technical condition and during time period $t\text{-}t_0$ each was affected by different overriding factors (varying with number and impact). Therefore, maintenance (involving predetermined activities) is often carried out with both high and low intensity rate.

Generally, the following maintenance intensity rates could be assumed:

- on-going maintenance (e.g. quarterly),
- average maintenance (e.g. annual).

Disadvantages of the resource based strategy

Main disadvantage of this strategy is the necessity to complete all scheduled maintenances, whilst the system could be in different technical conditions, due to varying intensity rate of overriding factors.

2.2. State based strategy

State based strategy involves continuous monitoring of technical condition of equipment and systems (e.g. using diagnostics subsystem [11]). Based on acquired information, rational maintenance measures are taken. This strategy does not rely on fixed dates of maintenance and repairs. All decisions concerning the need to carry them out are taken by the decision maker based on diagnostics information containing data on technical condition of equipment. This strategy should be applied to maintain systems required to be highly reliable due to health and safety reasons and (or) strategically key for the country (economy).

Disadvantages of the state based strategy

Main disadvantage of this strategy are higher costs to design and manufacture equipment since it needs to accommodate highly reliable diagnostics subsystems monitoring a wide range of parameters.

2.3. Mixed strategy

Between the resource based strategy and the state based strategy exist numerous intermediate solutions. They involve equipping systems using resource based strate-

gies with diagnostics subsystems supporting maintenance activities, e.g. diagnosing only some elements and devices constituting the system or monitoring only selected diagnostics signals. This strategy could be broken down into following types:
– sequence, i.e. only selected equipment sequence is diagnosed (e.g. chosen system elements),
– quasi-dynamic, i.e. only selected diagnostics signals are monitored, whose parameters impact dates and scope of maintenance.
– intermediate, i.e. the system is continuously diagnosed to the extent specified by economically viability.

Disadvantages of the mixed strategy

Main disadvantage of this strategy, similarly to the previous one, are higher costs of designing and manufacturing equipment since it needs to accommodate diagnostics subsystems.

2.4. Efficiency based strategy

Development of new technologies allows designers and manufacturers to introduce (in ever shorter time intervals) new types of elements and equipment. They usually would represent partial or holistic conceptual changes regarding:
– ergonomics,
– green credentials,
– energy efficiency,
– performance,
– efficiency,
– low maintenance costs.

Therefore, efficiency based maintenance strategy concerns equipment, whose ”relative” ageing outstrips their physical wear. When said equipment (despite its good technical condition) are withdrawn from used due to unsatisfactory efficiency or because failing to meet recently introduced criteria (e.g. compliance with international standards). For instance, a device might need replacing because it is not compatible with other new equipment (e.g. no wireless functionalities).

Disadvantages of the efficiency based strategy

The main disadvantage of this strategy is decommissioning technically capable equipment due to either its relative ageing rendering it obsolete compared to latest technical solutions or non-compliance with international standards.

2.5. Reliability based strategy

According to this strategy equipment shall be used until failure. Maintenance decisions are made based on different reliability parameters acquired through reliability research. This allows determining the weakest link the system. As of late, this strategy employs computer software for simulation forecasting, which allows to determine expected values of equipment lifetime until failure for different distributions of random variable.

Disadvantages of the reliability based strategy

Main disadvantage of this strategy is it could only be used for systems whose failure would not pose hazards to its surrounding environment and would not generate any additional failure-related costs (e.g. providing extra staff or replacement equipment).

3. Strategy Maximising Availability Rate

The maintenance strategies presented in the previous chapter did not explicitly factor in the availability rate. It definitely seems as if it should be factored into both the process of designing systems and their later maintenance. The availability rate is given by:

$$K_g = \frac{T_m}{T_m + T_n} \tag{1}$$

where: T_m – mean correct operation time between failures,

T_n – mean time to repair.

The given relationship shows that the system can be in one of two state (Fig. 2):
- usage state (S_0),
- repair state (S_1).

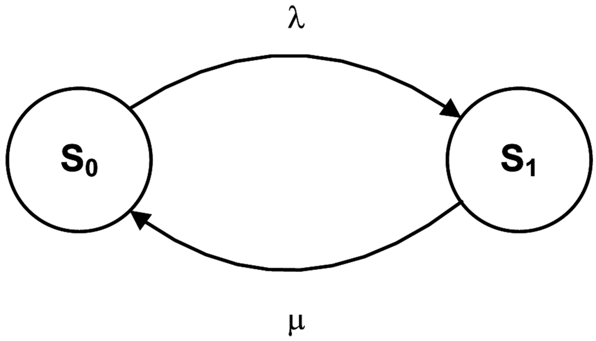

Fig. 2. Graph showing switching between usage and repair states
Denotations in figures: $\lambda-$ failure rate, $\mu-$ repair rate

Fig. 2 presents graph showing switching between states which does not include all possible and actual state. Hence the following states were added (Fig. 3):
- S_{001} state (basic servicing required by specification of type I inspection),
- S_{010} state (basic servicing required by specification of type II inspection),
- S_{011} state (basic servicing required by specification of type III inspection),

– . . . ,
– S_n state (basic servicing required by specification of n-th type inspection).

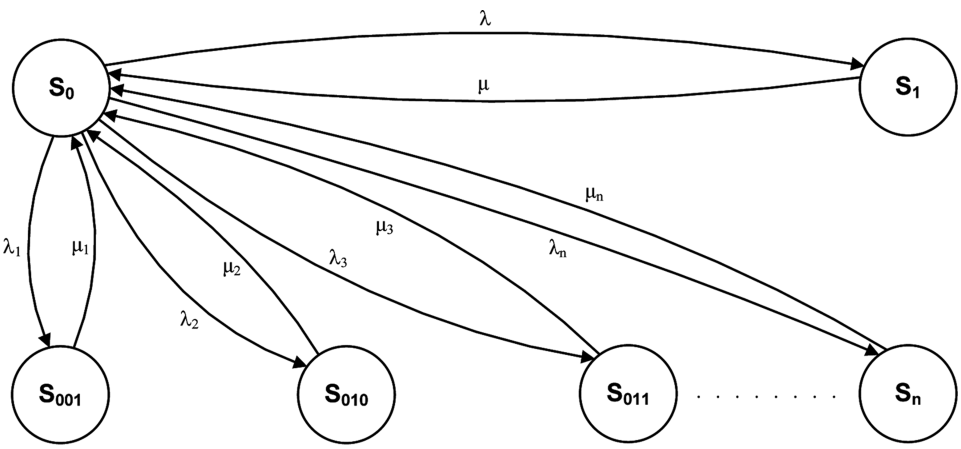

Fig. 3. Graph showing switching between usage state, repair state and I, II, III, ..., n-th type inspection state

Denotations in figures:
λ– failure rate,
μ– repair rate,
λ_1– I type inspection rate,
μ_1– I type routine maintenance rate,
λ_2– II type inspection rate,
μ_2– II type routine maintenance rate,
λ_3– III type inspection rate,
μ_3– III type routine maintenance rate,
λ_n – n-th type inspection rate,
μ_n – n-th type routine maintenance rate.

For the graph shown in figure 3 the following equations hold:

$$\begin{aligned}
&-\lambda \cdot P_0 + \mu \cdot P_1 - \lambda_1 \cdot P_0 + \mu_1 \cdot P_{001} - \lambda_2 \cdot P_0 + \mu_2 \cdot P_{010} - \lambda_3 \cdot P_0 + \\
&\mu_3 \cdot P_{011} + ... - \lambda_n \cdot P_0 + \mu_n \cdot P_n = 0 \\
&\lambda \cdot P_0 - \mu \cdot P_1 = 0 \\
&\lambda_1 \cdot P_0 - \mu_1 \cdot P_{001} = 0 \\
&\lambda_2 \cdot P_0 - \mu_2 \cdot P_{010} = 0 \\
&\lambda_3 \cdot P_0 - \mu_3 \cdot P_{011} = 0 \\
&... \\
&\lambda_n \cdot P_0 - \mu_n \cdot P_n = 0
\end{aligned} \quad (2)$$

In matrix notations they are given by:

$$\begin{bmatrix} -(\lambda+\lambda_1+\lambda_2+\lambda_3+\ldots+\lambda_n) & \mu & \mu_1 & \mu_2 & \mu_3 & \ldots & \mu_n \\ \lambda & -\mu & 0 & 0 & 0 & \ldots & 0 \\ \lambda_1 & 0 & -\mu_1 & 0 & 0 & \ldots & 0 \\ \lambda_2 & 0 & 0 & -\mu_2 & 0 & \ldots & 0 \\ \lambda_3 & 0 & 0 & 0 & -\mu_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ \lambda_n & 0 & 0 & 0 & 0 & 0 & -\mu_n \end{bmatrix} \cdot \begin{bmatrix} P_0 \\ P_1 \\ P_{001} \\ P_{010} \\ P_{011} \\ \ldots \\ P_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \ldots \\ 0 \end{bmatrix} \quad [3]$$

By rearranging we get:

$$P_1 = \frac{\lambda}{\mu} \cdot P_0$$

$$P_{001} = \frac{\lambda_1}{\mu_1} \cdot P_0$$

$$P_{010} = \frac{\lambda_2}{\mu_2} \cdot P_0$$

$$P_{011} = \frac{\lambda_3}{\mu_3} \cdot P_0$$

$$\cdots$$

$$P_n = \frac{\lambda_n}{\mu_n} \cdot P_0$$

Note:

$$P_0 + P_{001} + P_{010} + P_{011} + \ldots + P_n + P_1 = 1$$

Thus:

$$P_0 \cdot \left(1 + \frac{\lambda}{\mu} + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \ldots + \frac{\lambda_n}{\mu_n}\right) = 1 \quad (4)$$

$$K_{g1} = P_0 = \frac{1}{\left(1 + \frac{\lambda}{\mu} + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \ldots + \frac{\lambda_n}{\mu_n}\right)} \quad (5)$$

$$K_{g1} = P_0 = \frac{\mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \ldots \cdot \mu_n}{\begin{array}{l} \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \ldots \cdot \mu_n + \lambda \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \ldots \cdot \mu_n + \lambda_1 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot \ldots \cdot \mu_n + \\ + \lambda_2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot \ldots \cdot \mu_n + \lambda_3 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot \ldots \cdot \mu_n + \ldots + \\ + \lambda_n \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \ldots \cdot \mu_{n-1} \end{array}} \quad (6)$$

Let us introduce a coefficient, which will make rates $\lambda, \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ interlinked (should one increase, remaining will decrease). This coefficient will

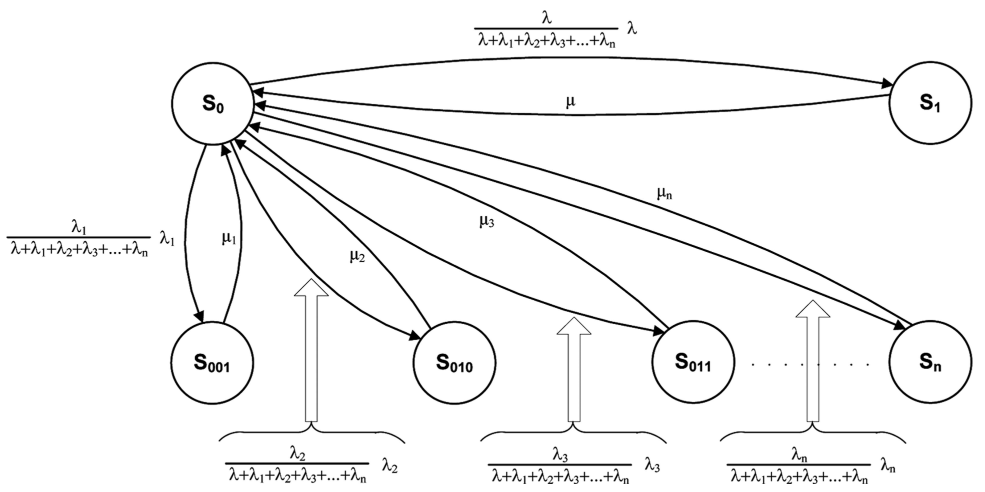

Fig. 4. Graph showing switching between usage state, repair state and I, II, III, ..., n-th type inspection state (the adjustment coefficient factored in)

represent the ratio of given transition rate to sum of all repair rates and I, II, II, ..., n-th type inspection. The graph shown in Figure 3 will become (Fig. 4):

As previously, we can write:

$$\begin{aligned}
&-\lambda \cdot \frac{\lambda}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 + \mu \cdot P_1 - \lambda_1 \cdot \frac{\lambda_1}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 + \\
&+\mu_1 \cdot P_{001} - \lambda_2 \cdot \frac{\lambda_2}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 \\
&+\mu_2 \cdot P_{010} - \lambda_3 \cdot \frac{\lambda_3}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 + \\
&+\mu_3 \cdot P_{011} + ... - \lambda_n \cdot \frac{\lambda_n}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 + \mu_n \cdot P_n = 0 \qquad [7] \\
&\lambda \cdot \frac{\lambda}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 - \mu \cdot P_1 = 0 \\
&\lambda_1 \cdot \frac{\lambda_1}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 - \mu_1 \cdot P_{001} = 0 \\
&\lambda_2 \cdot \frac{\lambda_2}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 - \mu_2 \cdot P_{010} = 0 \\
&\lambda_3 \cdot \frac{\lambda_3}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 - \mu_3 \cdot P_{011} = 0 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\lambda_n \cdot \frac{\lambda_n}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n} \cdot P_0 - \mu_n \cdot P_n = 0
\end{aligned}$$

In matrix notations they are given by:

$$\begin{bmatrix} -\frac{1}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot(\lambda^2+\lambda_1^2+\lambda_2^2+\lambda_3^2+...+\lambda_n^2) & \mu & \mu_1 & \mu_2 & \mu_3 & ... & \mu_n \\ \frac{\lambda}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\lambda & -\mu & 0 & 0 & 0 & ... & 0 \\ \frac{\lambda_1}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\lambda_1 & 0 & -\mu_1 & 0 & 0 & ... & 0 \\ \frac{\lambda_2}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\lambda_2 & 0 & 0 & -\mu_2 & 0 & ... & 0 \\ \frac{\lambda_3}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\lambda_3 & 0 & 0 & 0 & -\mu_3 & ... & 0 \\ ... & ... & ... & ... & ... & ... & 0 \\ \frac{\lambda_n}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\lambda_n & 0 & 0 & 0 & 0 & 0 & -\mu_n \end{bmatrix} \cdot \begin{bmatrix} P_0 \\ P_1 \\ P_{001} \\ P_{010} \\ P_{011} \\ ... \\ P_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ ... \\ 0 \end{bmatrix} \tag{8}$$

By rearranging we get:

$$P_1 = \frac{\lambda}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda}{\mu}\cdot P_0$$

$$P_{001} = \frac{\lambda_1}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda_1}{\mu_1}\cdot P_0$$

$$P_{010} = \frac{\lambda_2}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda_2}{\mu_2}\cdot P_0$$

$$P_{011} = \frac{\lambda_3}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda_3}{\mu_3}\cdot P_0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$P_n = \frac{\lambda_n}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda_n}{\mu_n}\cdot P_0$$

Note:

$$P_0 + P_{001} + P_{010} + P_{011} + ... + P_n + P_1 = 1$$

Thus:

$$\begin{aligned} &P_0\cdot(1+\frac{\lambda}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda}{\mu}+\frac{\lambda_1}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda_1}{\mu_1}+ \\ &+\frac{\lambda_2}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda_2}{\mu_2}+\frac{\lambda_3}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda_3}{\mu_3}+ \\ &+...+\frac{\lambda_n}{\lambda+\lambda_1+\lambda_2+\lambda_3+...+\lambda_n}\cdot\frac{\lambda_n}{\mu_n}) = 1 \end{aligned} \tag{9}$$

$$K_{g2} = P_0 = \frac{1}{\begin{array}{l}(1 + \frac{\lambda}{\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n} \cdot \frac{\lambda}{\mu} + \frac{\lambda_1}{\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n} \cdot \frac{\lambda_1}{\mu_1} + \\ + \frac{\lambda_2}{\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n} \cdot \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n} \cdot \frac{\lambda_3}{\mu_3} + \\ + ... + \frac{\lambda_n}{\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n} \cdot \frac{\lambda_n}{\mu_n})\end{array}} \tag{10}$$

$$K_{g2} = P_0 = \frac{(\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n}{\begin{array}{l}(\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ + \lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ + \lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ + ... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1}\end{array}} \tag{11}$$

The relationship obtained describes impact of pending I, II, III, ..., n-th type inspection rates on availability rate of given system (given failure rate and I, II, III, ..., n-th type routine maintenance rates are given). Should the function have a maximum, it is recommended to determine corresponding coordinates i.e. I, II, III, ..., n-th type inspection rate, since it will increase the availability rate. Those values would have been then optimum values, maximising the availability rate.

Let us investigate whether the function has a maximum. Derivative of the function is:

$$\frac{dP_0}{d\lambda_1} \quad , \quad \frac{dP_0}{d\lambda_2} \quad , \quad \frac{dP_0}{d\lambda_3} \quad , \quad \cdots \quad , \quad \frac{dP_0}{d\lambda_n}$$

$$\frac{dP_0}{d\lambda_1} = \frac{\begin{array}{l}\mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n \cdot \left[\begin{array}{l}(\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ + \lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ + \lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ + ... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1}\end{array}\right] - \\ - (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n \cdot \\ \cdot (\mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + 2 \cdot \lambda_1 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n)\end{array}}{\left[\begin{array}{l}(\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ + \lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ + \lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ + ... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1}\end{array}\right]^2} \tag{12}$$

$$\frac{dP_0}{d\lambda_2} = \frac{\begin{array}{l} \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n \cdot \begin{bmatrix} (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ +... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1} \end{bmatrix} - \\ - (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n \cdot \\ \cdot (\mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + 2 \cdot \lambda_2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n) \end{array}}{\begin{bmatrix} (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ +... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1} \end{bmatrix}^2} \tag{13}$$

$$\frac{dP_0}{d\lambda_3} = \frac{\begin{array}{l} \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n \cdot \begin{bmatrix} (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ +... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1} \end{bmatrix} - \\ - (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n \cdot \\ \cdot (\mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + 2 \cdot \lambda_3 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n) \end{array}}{\begin{bmatrix} (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ +... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1} \end{bmatrix}^2} \tag{14}$$

....

$$\frac{dP_0}{d\lambda_n} = \frac{\begin{array}{l} \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n \cdot \begin{bmatrix} (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ +... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1} \end{bmatrix} - \\ - (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n \cdot \\ \cdot (\mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + 2 \cdot \lambda_n \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1}) \end{array}}{\begin{bmatrix} (\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ +... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1} \end{bmatrix}^2} \tag{15}$$

A condition necessary, for the function $P_0(\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n)$ to have extremum at $P_0\left(\lambda_{1optym}, \lambda_{2optym}, \lambda_{3optym}, ..., \lambda_{noptym}\right)$, is that first partial derivatives of the function at that point has to equal zero, i.e.:

$$\begin{cases} \frac{dP_0}{d\lambda_1}(\lambda_{1optym}, \lambda_{2optym}, \lambda_{3optym}, ..., \lambda_{noptym}) = 0 \\ \frac{dP_0}{d\lambda_2}(\lambda_{1optym}, \lambda_{2optym}, \lambda_{3optym}, ..., \lambda_{noptym}) = 0 \\ \frac{dP_0}{d\lambda_3}(\lambda_{1optym}, \lambda_{2optym}, \lambda_{3optym}, ..., \lambda_{noptym}) = 0 \\ .... \\ \frac{dP_0}{d\lambda_n}(\lambda_{1optym}, \lambda_{2optym}, \lambda_{3optym}, ..., \lambda_{noptym}) = 0 \end{cases} \quad (16)$$

Herein presented maintenance analysis returns a criterion essential for evaluating a maintenance process. It is related to the availability rate, which should be maximised provided taken initial conditions:

– failure rate λ,
– repair rate μ,
– I type routine maintenance rate μ_1,
– II type routine maintenance rate μ_2,
– III type routine maintenance rate μ_3,
– . . . ,
– n type routine maintenance rate μ_n.

For presented method the criterion function becomes:

$$K_g(\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n) = \frac{(\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n}{\begin{gathered}(\lambda + \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_1^2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_n + \\ +\lambda_2^2 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot ... \cdot \mu_n + \lambda_3^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 \cdot ... \cdot \mu_n + \\ +... + \lambda_n^2 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot ... \cdot \mu_{n-1}\end{gathered}} \quad (17)$$

Sought after are:

– I type inspection rate λ_1,
– II type inspection rate λ_2,
– III type inspection rate λ_3,
– . . . ,
– n type inspection rate λ_n,

for which non-linear criterion function yields a maximum value:

$$\max_{\{(\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n)\}} K_g(\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n)$$

provided constraints:

$$\lambda_1 \in \langle 0, 1\rangle$$
$$\lambda_2 \in \langle 0, 1\rangle$$
$$\lambda_3 \in \langle 0, 1\rangle$$
$$\ldots$$
$$\lambda_n \in \langle 0, 1\rangle$$

4. Practical Application of Maintenance Strategy Maximising Availability Rate

Presented maintenance strategy maximising availability rate allows factoring in n types of inspections. In order to affirm correctness of conducted deliberations, calculations were carried out for an example where two types of inspections were assumed. Hence the transition graph would become as shown in Fig. 5.

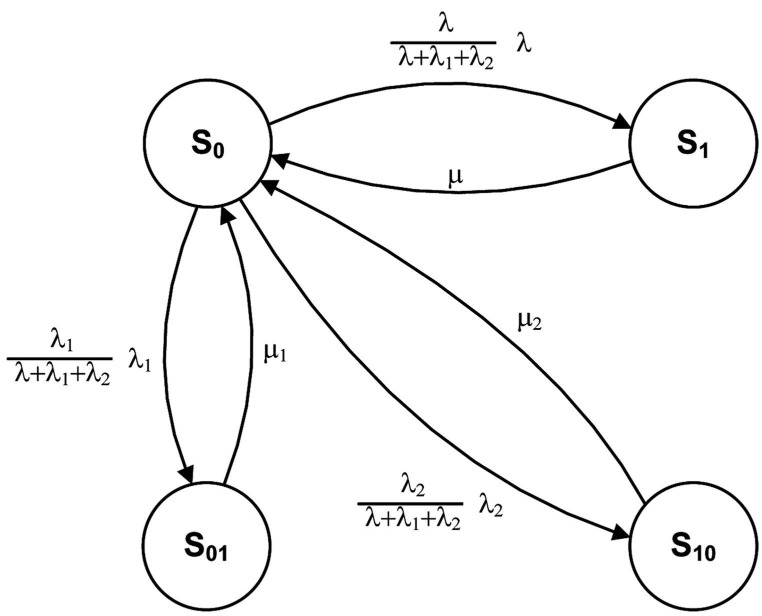

Fig. 5. Graph showing switching between usage state (S_0), repair state (S_1), I inspection state (S_{01}) and II inspection state (S_{10}) (adjustment coefficient)

For the graph shown in figure 5 the following relationship was obtained:

$$K_{g\_przyklad} = P_0 = \frac{(\lambda + \lambda_1 + \lambda_2) \cdot \mu \cdot \mu_1 \cdot \mu_2}{(\lambda + \lambda_1 + \lambda_2) \cdot \mu \cdot \mu_1 \cdot \mu_2 + \lambda^2 \cdot \mu_1 \cdot \mu_2 + \lambda_1^2 \cdot \mu \cdot \mu_2 + \lambda_2^2 \cdot \mu \cdot \mu_1}$$

A condition necessary, for the function $K_{g\_przyklad}(\lambda_1, \lambda_2)$ to have extreme at $K_{g\_przyklad}\left(\lambda_{1optym}, \lambda_{2optym}\right)$, is that first partial derivatives of the function at that point has to equal zero, i.e.:

$$\begin{cases} \dfrac{dK_{g\_przyklad}}{d\lambda_1}(\lambda_{1optym}, \lambda_{2optym}) = 0 \\ \dfrac{dK_{g\_przyklad}}{d\lambda_2}(\lambda_{1optym}, \lambda_{2optym}) = 0 \end{cases}$$

Due to substantial mathematical complexity, the MathCAD software was employed. It enabled to present the function graphically $K_{g\_przyklad}(\lambda_1, \lambda_2)$. Hence coordinates $\left(\lambda_{1optym}, \lambda_{2optym}\right)$ of the maximum could be established. This has been illustrated by below example.

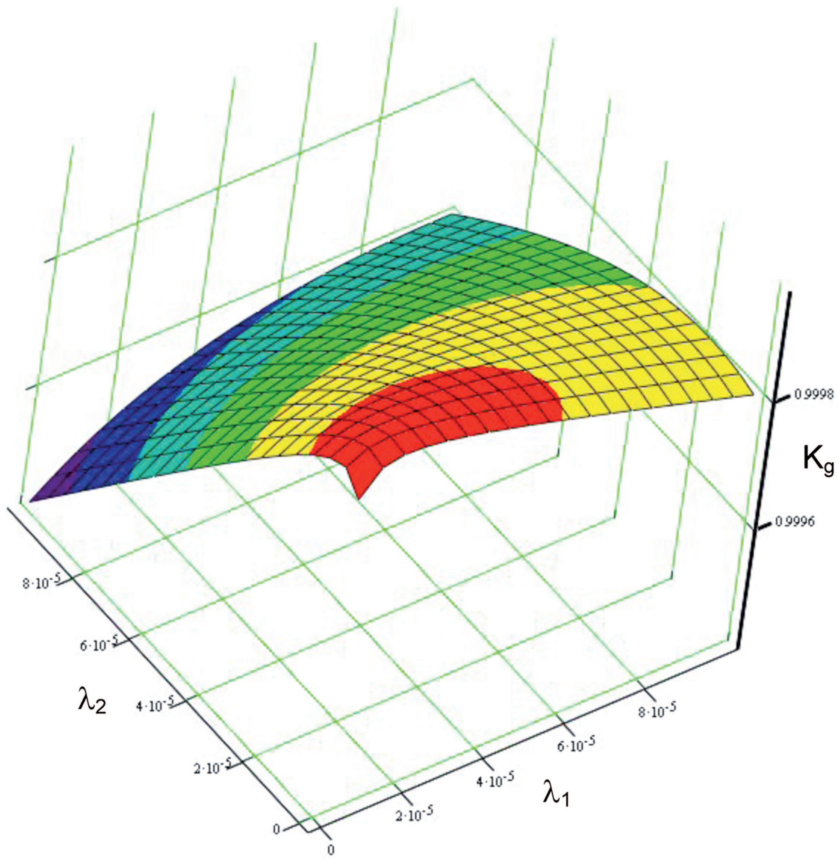

Fig. 6. Relationship between availability rate K_g as function of I type inspection coefficient λ_1 and II type inspection coefficient λ_2

Let us assume that:

– investigation time $t_B = 8760\,[h]$,
– failure rate $\lambda = 5,855399 \cdot 10^{-6} \left[\frac{1}{\mathrm{h}}\right]$ (representing system whose reliability is 0.95; exponential distribution),
– repair rate $\mu = 0,0666 \left[\frac{1}{\mathrm{h}}\right]$ (representing repair time of 15 [h]),
– I type routine maintenance rate $\mu_1 = 0,5 \left[\frac{1}{\mathrm{h}}\right]$ (representing inspection time of 2 [h]),
– II type routine maintenance rate $\mu_2 = 0,1666 \left[\frac{1}{\mathrm{h}}\right]$ (representing inspection time of 6 [h]).

For the taken assumptions a graph was plotted, which has been presented in Fig. 6 (view from x-axis of the coordinate system) and Fig. 7 (”top view” of contour line).

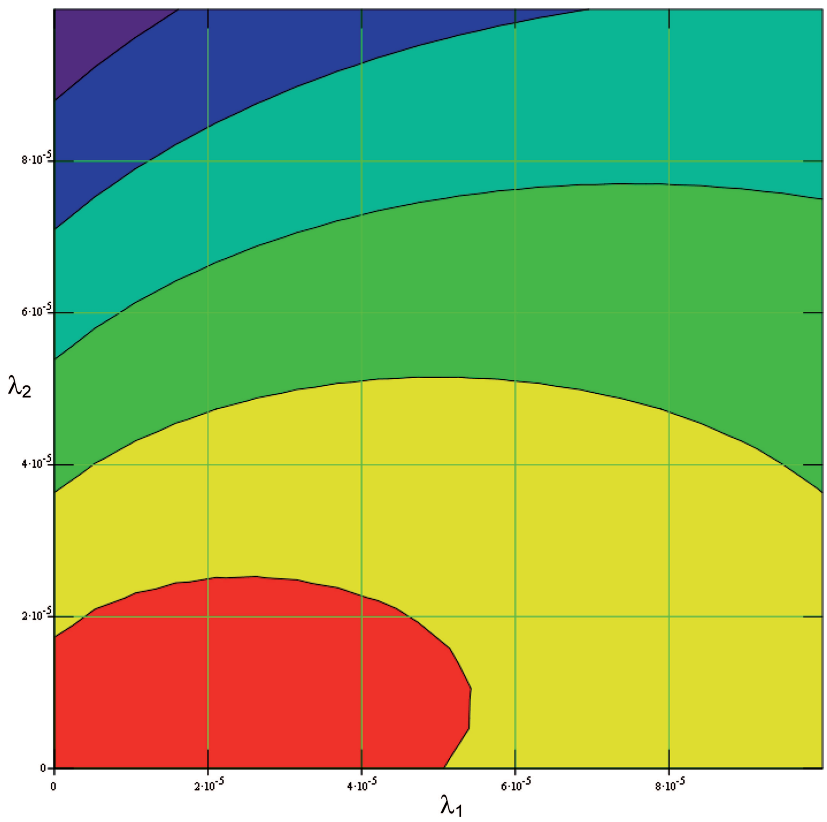

Fig. 7. Relationship between availability rate K_g as function of I type inspection coefficient λ_1 and II type inspection coefficient λ_2 (”top view” of contour line)

Coordinates $\left(\lambda_{1optym}, \lambda_{2optym}\right)$ of the maximum are as follows:

$$\lambda_{1optym} = 1,017 \cdot 10^{-5} \left[\frac{1}{\mathrm{h}}\right]$$
$$\lambda_{2optym} = 3,392 \cdot 10^{-6} \left[\frac{1}{\mathrm{h}}\right]$$

In order to determine said coordinates, Fig. 6 and 7 were used. The availability rate is maximum at those coordinates and equals to K_g = 0,99995931.

5. Conclusions

The presented maintenance strategy maximising the availability rate enables optimising that parameter through obtaining information about inspection rate of I, II, III, ..., n-th type. Nevertheless, provided rapid development of electronic systems employed in transport and the diagnostics subsystem they use, it is fair to say the trend in designing tends towards developing and implementing systems with diagnosing and therapeutic capabilities. They will have overseen the system and taken ever complex (factoring in the reliability theory, the maintenance theory) therapeutic measures, preventing the system from collapsing into the state of reached operational capability.

The presented method of optimising the maintenance process requires knowledge on theoretical notions behind the reliability and maintenance theory. Hence, there is a need to develop a computer application which would determine optimum inspection rates. A solution of this calibre would have enabled the users and maintenance officers to quickly and correctly deploy the developed method.

References

1. Będkowski L., Dąbrowski T.: Basics of maintenance, vol. II Basic of operational reliability. Warsaw Academy of Technology, Warsaw 2006.
2. Dyduch J., Moczarski J.: Basics of maintaining the train traffic control system. Publishing House of Radom University of Technology, Radom 2009.
3. Dyduch J., Paś J., Rosiński A.: Basics of maintaining electronic transport systems. Publishing House of Radom University of Technology, Radom 2011.
4. Dyduch J., Rosiński A.: Selected aspects of maintenance and reliability of security systems. Scientific papers of Radom University of Technology 1(15) 2002, Radom 2002.
5. Jaźwiński J., Ważyńska-Fiok K.: Safety of systems. PWN, Warsaw 1993.
6. Rosiński A., Dąbrowski T.: Modelling reliability of power supply units. XL Winter School of Reliability, Szczyrk 2012.
7. Rosiński A.: Reliability analysis of the electronic protection systems with mixed m–branches reliability structure. International Conference European Safety and Reliability (ESREL 2011), Troyes, France 2011.
8. Rosiński A.: Reliability analysis of transport structures of surveillance systems. Scientific papers of Radom University of Technology 3(23) 2005, Radom 2005.

9. Rosiński A.: Design of the electronic protection systems with utilization of the method of analysis of reliebility structures. Nineteenth International Conference On Systems Engineering (ICSEng 2008), Las Vegas, USA 2008.
10. Rosiński A.: Design of the transport supervision systems with utilization of the method of determination of reliability structures. TRANSCOM 2007, Żilina, Słowacja.
11. Rosiński A.: Diagnostics of electronic security systems. VII National Conference: "Technical diagnostics of equipment and systems – DIAG'2009", Ustroń 2009.
12. Rosiński A.: Exploational strategies in transport supervision systems. 10th International Conference „Computer Systems Aided Science, Industry and Transport" TRANSCOMP 2006, Zakopane 2006.
13. Rosiński A.: Exploitation strategies of monitoring transport systems with the regard of economic conditions. 4th International Congress on Technical Diagnostics, DIAGNOSTICS 2008, Olsztyn 2008.
14. Rosiński A.: Method for optimising routine inspections of transport surveillance systems factoring in costings. Diagnostics no. 1(45)/2008.
15. Rosiński A.: Method for optimising routine inspections of transport surveillance systems. Diagnostics no. 2(38)/2006.
16. Rosiński A.: Method for selecting maintenance strategy in transport surveillance systems – doctoral dissertation. Warsaw University of Technology, Faculty of Transport, Warsaw 2007.
17. Rosiński A.: Reliability analysis of the electronic protection systems with mixed – three branches reliability structure. International Conference European Safety and Reliability (ESREL 2009), Prague, Czech Republic 2009.
18. Rosiński A.: Maintenance strategy in transport surveillance systems. Scientific papers transport issue 62, Publishing House of Warsaw University of Technology, Warsaw 2007.
19. Siergiejczyk M., Rosiński A.: Reliability analysis of electronic protection systems using optical links. The monograph „Dependable Computer Systems", editors: Wojciech Zamojski, Janusz Kacprzyk, Jacek Mazurkiewicz, Jarosław Sugier i Tomasz Walkowiak, given as the monographic publishing series – „Advances in intelligent and soft computing", Vol. 97. The publisher: Springer-Verlag, Berlin Heidelberg (2011).
20. Siergiejczyk M., Rosiński A.: Reliability analysis of power supply systems for devices used in transport telematic systems. The monograph „Modern Transport Telematics", editors: Jerzy Mikulski, given as the monographic publishing series – „Communications in Computer and Information Science", Vol. 239. The publisher: Springer-Verlag, Berlin Heidelberg (2011).
21. Siergiejczyk M., Rosiński A.: Optimisation of transport telematics electronic systems operational process. „Polish Journal Of Environmental Studies". Stud. Vol. 20, No. 5A (2011).
22. Siergiejczyk M.: Operational effectiveness of transport telematic systems. Scientific papers of Warsaw University of Technology, Transport, No. 67, Warsaw 2009.
23. Smalko Z.: Basics of technical maintenance of vehicles. OWPW, Warsaw 1998.
24. Wawrzyński W.: Security of control systems used in transport. Institute of Maintenance Technology, Radom 2004.
25. Ważynska-Fiok K.: Basic of maintenance and reliability theory for transport systems. WPW, Warsaw 1993.
26. Woropay M.: Basics of rational machinery maintenance. ATR, Bydgoszcz 1996.
27. Żółtowski B., Niziński S.: Modelling machinery maintenance processes. ATR, Bydgoszcz 2002.