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Availability Control for Means of Transport in Decisive Semi-Markov Models of Exploitation Process

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Abstract

The issues presented in this research paper refer to problems connected with the control process for exploitation implemented in the complex systems of exploitation for technical objects. The article presents the description of the method concerning the control availability for technical objects (means of transport) on the basis of the mathematical model of the exploitation process with the implementation of the decisive processes by semi-Markov. The presented method means focused on the preparing the decisive for the exploitation process for technical objects (semi-Markov model) and after that specifying the best control strategy (optimal strategy) from among possible decisive variants in accordance with the approved criterion (criteria) of the activity evaluation of the system of exploitation for technical objects. In the presented method specifying the optimal strategy for control availability in the technical objects means a choice of a sequence of control decisions made in individual states of modelled exploitation process for which the function being a criterion of evaluation reaches the extreme value. In order to choose the optimal control strategy the implementation of the genetic algorithm was chosen. The opinions were presented on the example of the exploitation process of the means of transport implemented in the real system of the bus municipal transport. The model of the exploitation process for the means of transports was prepared on the basis of the results implemented in the real transport system. The mathematical model of the exploitation process was built taking into consideration the fact that the model of the process constitutes the homogenous semi-Markov process.

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1. Introduction

Providing high effectiveness of activity for complex exploitation systems of the technical objects is possible only in case when the control decisions are sensible and made by the decision-makers for the system. In the systems in which the implemented exploitation process for technical objects is complex the choice of rational control decisions among from possible decisive variants is a difficult and complicated issue. In the real complex exploitation systems for the technical objects the process of making control decisions should be implemented with the use of proper methods and mathematical tools. This means no in the "intuitive" system based only on knowledge and experience of the decision-makers. The implementation of proper mathematical methods to control the exploitation process makes the choice of sensible control decisions easier. In this way the proper and effective implementation of the tasks ascribed to the system is provided.

In order to provide the proper decisive process the assisting tools are implemented. They include various types of decisive models where the key element is the mathematical model of the exploitation process for the exploitation of technical objects. The mathematical preparation for the model of the exploitation process for technical objects (means of transport) provides easier analysis of the process which is the basis for the evaluation and sensible control of the process [6, 8, 11]. Due to the random character of the factors having an impact on the course and effectiveness of the exploitation process for the means of transport implemented in the complex system the most mathematical modelling of the exploitation process use stochastic versions. Random process includes a wide implementation of Markov and semi-Markov process for modelling the exploitation process for technical objects [1, 7, 9, 14]. The implementation of the semi-Markov process allows to construct and analyze the mathematical model for the exploitation process. In case the variables characteristic for the states of the process are described in different ways than the exponential one.

The decisive (control) semi-Markov processes constitute a convenient mathematical tool which implementation makes the complicated process of making sensible control decisions easier in the complex exploitation systems for technical objects. In the literature concerning he issues of control for exploitation processes in technical objects there is a lot papers concerning both the theoretical description as well as examples of practical implementations of control semi-Markov processes [1, 2, 7, 15]. The research paper presents an example of the implementation of control semi-Markov processes to control availability in the exploitation system for the means of transport.

2. Implementation of Decisive Semi-Markov Processes to Control Availability of the Means of Transport

Due to the needs the decisive semi-Markov processes can be implemented for the mathematical formulating and solving a wide spectrum of problems referring to control the exploitation process in technical objects, including the economic analysis, risk managing and performance safety for complex technical systems as well as control of availability and reliability of operated technical objects (means of transport) [1, 2, 7, 13].

The decisive semi-Markov process is a stochastic process $\{X(t): t \ge 0\}$, the implementation of which depends on the decisions made at the beginning of the process t_0 and at the moments of changing the process $t_1, t_2, \ldots, t_n, \ldots$ At work it is assumed that the analyzed semi-Markov process possess a limited number of states $i = 1, 2, \ldots, m$. Then:

$$D_{i} = \left\{ d_{i}^{(1)}(t_{n}), d_{i}^{(2)}(t_{n}), ..., d_{i}^{(k)}(t_{n}) \right\}$$
(1)

means a set of all possible control decisions which can be implemented in *i*-state of the process at the moment of t_n , where $d_i^{(k)}(t_n)$ means *k*-control decision made in *i*-state of the process, at the moment of t_n . In case of implementation of the decisive semi-Markov processes making the decision at the moment of t_n , *k*-controlling decision in *i*-state of the process means a choice of *i*-verse of the core of the matrix from the following set:

$$\left\{Q_{ij}^{(k)}(t): t \ge 0, \ d_i^{(k)}(t_n) \in D_i, \ i, j \in S\right\},\tag{2}$$

where

$$Q_{ij}^{(k)}(t) = p_{ij}^{(k)} \cdot F_{ij}^{(k)}(t).$$
(3)

The choice of the *i*-verse of the core of the process specifies the probabilistic mechanism of evolution of the process in the period of time $\langle t_n, t_{n+1} \rangle$. This means that for the semi-Markov process, in case of the change of the state of the process from one into *i*-one (entry to the *i*-state of the process) at the moment t_n , the decision is made $d_i^{(k)}(t_n) \in D_i$ and according to the schedule $\left(p_{ij}^{(k)} : j \in S\right) j$ -state of the process is generated, which is entered at the moment of t_{n+1} . At the same time, in accordance with the schedule specified by the distributor $F_{ij}^{(k)}(t)$, the length of the period of time is generated $\langle t_n, t_{n+1} \rangle$ to leave the *i*-state of the process, when the next state is the *j*-state. Then as the strategy we understand the δ sequence, where the words are the vectors, comprising of the decision $d_i^{(k)}(t_n)$ made in the following moments of the t_n changes of the state of the process X(t):

$$\delta = \left\{ \left[d_1^{(k)}(t_n), d_2^{(k)}(t_n), ..., d_m^{(k)}(t_n) \right] : n = 0, 1, 2, ... \right\}$$
(4)

The δ strategy is named a stationary strategy in case when the decisions made in the following stages of the process do not depend on the moment t_n , in which they

are made which means in $d_i^{(k)}(t_n) = d_i^{(k)}$. In this case the semi-Markov process is the homogenous process and the control strategy is presented in the following way:

$$\delta = \left[d_1^{(k)}, d_2^{(k)}, ..., d_m^{(k)} \right]$$
(5)

The choice of the proper control strategy δ named the optimal strategy δ^* refers to the situation when the function being the criterion of the choice of the optima strategy takes an extreme value (minimal or maximal). In case of implementing decisive semi-Markov processes to control availability of the technical objects (means of transport) the criteria function can be the function describing availability of individual technical object G^{OT} . The choice of the optimal strategy δ^* is made on the basis of the following criterion:

$$G^{OT}(\delta^*) = \max_{\delta} \left[G^{OT}(\delta) \right]$$
(6)

3. Availability of the Means of Transport Established on the Basis of the Semi-Markov Model of Exploitation Process

In order to implement the decisive semi-Markov processes to control availability of the means of transport, operated in the analyzed system of the bus municipal transport the mathematical model of the exploitation process was prepared. The mathematical model of the exploitation process for the means of transport was extended on the basis of the incidental model of the exploitation process with the use of the theory of the semi-Markov's systems. The mathematical model for the exploitation process constitutes the basis for evaluation availability of the implemented means of transport.

3.1. Incidental model of the exploitation process for the means of transport

The incidental model of the exploitation process was extender on the basis of the analysis of the space of states and exploitative events concerning the operated means of transport in the real, analyzed municipal bus transport system. In the result of the identification of the analyzed transport system and the implemented, multi-stage exploitation process the essential exploitation states of the process and possible changes from one into another were determined. The prepared incidental model of the exploitation process for the means of transport is presented by means of a graph drawn in the Figure 1.

The analyzed model of the exploitation process distinguishes the following exploitation states for the means of transport:

Availability Control for Means of Transport in Decisive Semi-Markov...

- S_1 waiting for the implementation of the task in the bus depot,
- S_2 preparing the bus depot for without the loss of the bus ride,
- S_3 implementation of the transport task,
- S₄ damage during the implementation of the transport task,
- S₅ diagnosis by technical emergency team,
- S₆ repair made by the technical emergency team without looping the ride,
- S₇ repair made by the technical emergency team with a loss of a ride,
- S_8 waiting for the implementation of the task after the repair made by the technical emergency team,
- S₉ emergency exit,
- S₁₀ waiting for an entry to the sub-system for providing qualification,
- S_{11} tanking up,
- S₁₂ implementation of service on the operating day,
- S_{13} implementation of the periodical technical service,
- S₁₄ diagnosis before the repair in the subsystem for providing qualification,
- S_{15} repair in the subsystem for providing qualification,
- S_{16} diagnosis after the repair in the subsystem for providing qualification.

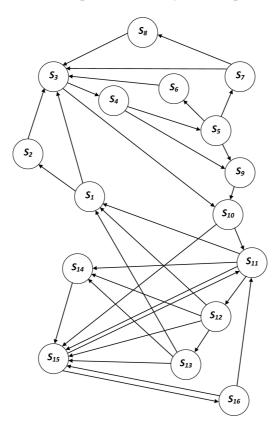


Fig. 1. Graph orientated to imitate the exploitation process for the means of transport

3.2. Semi-Markov model for the exploitation process of the means of transport

The semi-Markov process X(t) is the process where times between the following changes of the states of the process have numerous layout of probability and moving from one state into another depends only on the current state of the process.

During the implementation of the semi-Markov processes to the mathematical modelling of the exploitation process for the means of transport the following guidelines were approved:

- the modelled exploitation process possess a limited number of states S_i , i = 1, 2, ..., 16,
- if the technical object at the moment t is in the state S_i , to X(t) = i, where i = 1, 2, ..., 16,
- the random X(t) being the mathematical model of the exploitation process is the homogeneous process,
- at the moment t = 0 the process is in the state S_1 (S_1 is the initial state).

The homogeneous semi-Markov process is clearly specified when the initial layout of the process and the core are provided. On the basis of the approved guidelines and on the basis of the graph prepared according to the Figure 1, the initial layout has the following form:

$$p_i(0) = \begin{cases} 1, \text{ if } i = 1\\ 0, \text{ if } i \neq 1 \end{cases},$$
(7)

where:

$$p_i(0) = P\{X(0) = i\}, \quad i = 1, 2, ..., 16,$$
(8)

and the core of the process possess the following form Q(t):

	0	$Q_{1,2}(t)$	$Q_{1,3}(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	$Q_{2,3}(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	$Q_{3,4}(t)$	0	0	0	0	0	$Q_{3,10}(t)$	0	0	0	0	0	0	
Q(t) =	0	0	0	0	$Q_{4,5}(t)$	0	0	0	$Q_{4,9}(t)$	0	0	0	0	0	0	0	
	0	0	0	0	0	$Q_{5,6}(t)$	$Q_{5,7}(t)$	0	$Q_{5,9}(t)$	0	0	0	0	0	0	0	(9)
	0	0	$Q_{6,3}(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0	(\mathcal{I})
	0	0	$Q_{7,3}(t)$	0	0	0	0	$Q_{7,8}(t)$	0	0	0	0	0	0	0	0	
	0	0	$Q_{8,3}(t)$	0	0	0	0	0	0	0	0	0	0	0	0	0	
Q(t)-	0	0	0	0	0	0	0	0	0	$Q_{9,10}(t)$	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	$Q_{10,11}(t)$	0	0	0	$Q_{10,15}(t)$	0	
	$Q_{11,1}(t)$	0	0	0	0	0	0	0	0	0	0	$Q_{11,12}(t)$	0	$Q_{11,14}(t)$	$Q_{11,15}(t)$	0	
	$Q_{12,1}(t)$	0	0	0	0	0	0	0	0	0	0	0	$Q_{12,13}(t)$	$Q_{12,14}(t)$	$Q_{12,15}(t)$	0	
	$Q_{13,1}(t)$	0	0	0	0	0	0	0	0	0	0	0	0	$Q_{13,14}(t)$	$Q_{13,15}(t)$	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$Q_{14,15}(t)$	0	
	0	0	0	0	0	0	0	0	0	0	$Q_{15,11}(t)$	0	0	0	0	$Q_{15,16}(t)$	
	0	0	0	0	0	0	0	0	0	0	$Q_{16,11}(t)$	0	0	0	$Q_{16,15}(t)$	0	

where:

$$Q_{ij}(t) = P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n \le t | X(\tau_n) = i\}, \quad i, j = 1, 2, ..., 16$$
(10)

means that the state of the semi-Markov process and its duration time depends only on the previous state and does not depend of the previous states and their duration times where $\tau_1, \tau_2, \ldots, \tau_n$ can be any moments of time, such as $\tau_1 < \tau_2 < \ldots < \tau_n$ and

$$Q_{ij}(t) = p_{ij} \cdot F_{ij}(t) \tag{11}$$

where:

 $p_{ij} = \lim_{t \to \infty} p_{ij}(t)$ – conditional probability to change the state from S_i into S_j , $F_{ij}(t)$ – the distribution function of the random variable Θ_{ij} meaning the duration time of the state S_i , on condition that the next stage will be the stage S_j , and the presented relationship:

$$F_{ij}(t) = P\{\tau_{n+1} - \tau_n \le t | X(\tau_n) = i, X(\tau_{n+1}) = j\}, \quad i, j = 1, 2, ..., 16$$
(12)

Therefore, in order to determine the boundary timetable for the semi-Markov process the following matrixes were constructed: P of probabilities for the states of changes and Θ conditional duration times of the states of the process X(t):

	0	$p_{1,2}$	<i>p</i> _{1,3}	0	0	0	0	0	0	0	0	0	0	0	0	0	
<i>P</i> =	0	0	<i>p</i> _{2,3}	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	<i>p</i> _{3,4}	0	0	0	0	0	<i>p</i> _{3,10}	0	0	0	0	0	0	
	0	0	0	0	$p_{4,5}$	0	0	0	$p_{4,9}$	0	0	0	0	0	0	0	
	0	0	0	0	0	$p_{5,6}$	$p_{5,7}$	0	$p_{5,9}$	0	0	0	0	0	0	0	
	0	0	$p_{6,3}$	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	$p_{7,3}$	0	0	0	0	$p_{7,8}$	0	0	0	0	0	0	0	0	
	0	0	$p_{8,3}$	0	0	0	0	0	0	0	0	0	0	0	0	0	(13)
Γ –	0	0	0	0	0	0	0	0	0	$p_{9,10}$	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	$p_{10,11}$	0	0	0	$p_{10,15}$	0	
	$p_{11,1}$	0	0	0	0	0	0	0	0	0	0	$p_{11,12}$	0	$p_{11,14}$	$p_{11,15}$	0	
	$p_{12,1}$	0	0	0	0	0	0	0	0	0	0	0	$p_{12,13}$	$p_{12,14}$	$p_{12,15}$	0	
	<i>p</i> _{13,1}	0	0	0	0	0	0	0	0	0	0	0	0	$p_{13,14}$	<i>p</i> _{13,15}	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	<i>p</i> _{14,15}	0	
	0	0	0	0	0	0	0	0	0	0	$p_{15,11}$	0	0	0	0	$p_{15,16}$	
	0	0	0	0	0	0	0	0	0	0	$p_{16,11}$	0	0	0	$p_{16,15}$	0	

		-	-	0	0	0	0	0	0	0	0	0	0	0	0	-	í
Θ=	0	$\overline{\mathcal{O}}_{\!\!\!1,2}$	$\overline{\varTheta}_{1,3}$	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	$\overline{\Theta}_{2,3}$	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	$\overline{\varTheta}_{3,4}$	0	0	0	0	0	$\overline{\mathcal{O}}_{3,10}$	0	0	0	0	0	0	
	0	0	0	0	$\overline{\varTheta}_{4,5}$	0	0	0	$\overline{\varTheta}_{4,9}$	0	0	0	0	0	0	0	
	0	0	0	0	0	$\overline{\varTheta}_{5,6}$	$\overline{\varTheta}_{5,7}$	0	$\overline{\varTheta}_{5,9}$	0	0	0	0	0	0	0	
	0	0	$\overline{\mathcal{O}}_{6,3}$	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	$\overline{O}_{7,3}$	0	0	0	0	$\overline{\varTheta}_{7,8}$	0	0	0	0	0	0	0	0	(1.0)
	0	0	$\overline{\Theta}_{8,3}$	0	0	0	0	0	0	0	0	0	0	0	0	0	(14)
6 -	0	0	0	0	0	0	0	0	0	$\overline{\mathcal{O}}_{9,10}$	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	$\overline{\Theta}_{10,11}$	0	0	0	$\overline{\Theta}_{10,15}$	0	
	$\overline{O}_{11,1}$	0	0	0	0	0	0	0	0	0	0	$\overline{\Theta}_{11,12}$	0	$\overline{\Theta}_{11,14}$	$\overline{\Theta}_{11,15}$	0	
	$\overline{\Theta}_{12,1}$	0	0	0	0	0	0	0	0	0	0	0	$\overline{\Theta}_{12,13}$	$\overline{\Theta}_{12,14}$	$\overline{\Theta}_{12,15}$	0	
	$\overline{\Theta}_{13,1}$	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{\varTheta}_{13,14}$	$\overline{\Theta}_{13,15}$	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\Theta_{14,15}$	0	
	0	0	0	0	0	0	0	0	0	0	$\overline{\Theta}_{15,11}$	0	0	0	0	$\overline{\Theta}_{15,16}$	
	0	0	0	0	0	0	0	0	0	0	$\overline{\varTheta}_{16,11}$	0	0	0	$\overline{\Theta}_{16,15}$	0	

Therefore, the boundary probability p_i^* for staining in the states of the semi-Markov processes can be determined on the basis of the boundary statement for the semi-Markov process [3, 6], in accordance with the following pattern:

$$p_i^* = \lim_{t \to \infty} p_i(t) = \frac{\pi_i \cdot E(\Theta_i)}{\sum\limits_{i \in S} \pi_i \cdot E(\Theta_i)}$$
(15)

where:

probabilities π_i , $i \in S$ constitute the stationary layout of the implemented Markov's chain in the process which fulfils the system of linear equations:

$$\sum_{i \in S} \pi_i \cdot p_{ij} = \pi_j, \quad j \in S, \sum_{i \in S} \pi_i = 1$$
(16)

 $E(\Theta_i), i \in S$ – values for unconditional expected values for duration times of the states of the process:

$$E(\Theta_i) = \int_0^\infty t d\left[\sum_j F_{ij}(t)\right], \quad i, j = 1, 2, ..., 16$$
(17)

estimated for the average values $\overline{\Theta_i}$, on the basis of the *P* matrix and Θ matrix, according to the following relationship:

$$\overline{\Theta_i} = \sum_i p_{ij} \cdot \overline{\Theta}_{ij}, \quad i, j = 1, 2, ..., 16$$
(18)

In order to determine availability for the technical objects (means of transport) on the basis of the semi-Markov model of the exploitation process the exploitation states of the technical object should be divided into the states of availability S_G and non-availability S_{NG} of the object for the assigned task. The presented model distinguishes the following states of availability for the technical object:

 S_1 – waiting for the implementation of the task in the bus depot,

 S_2 – preparing the bus depot for without the loss of the bus ride,

S₃ – implementation of the transport task,

S₆ – repair made by the technical emergency team without looping the ride,

 S_8 – waiting for the implementation of the task after the repair made by the technical emergency team.

In such case availability of the individual technical object is determined as a sum of boundary probabilities p_i^* of staying in the states of the modelled process, belonging to the set of the availability states S_G [3, 10, 14]:

$$G^{OT} = \sum_{i} p_i^*, \text{ for } S_i \in S_G$$
(19)

for the modelled exploitation process of the means of transport is amounts to the following:

$$G^{OI} = p_1^* + p_2^* + p_3^* + p_6^* + p_8^*$$
(20)

4. Choosing the Optimal Strategy for Controlling Availability of the Means of Transport

Taking into consideration the fact that in each modelled exploitation process sit was possible to implement one of two decisions, the number of control strategies to be implemented foe the presented model of the exploitation process of the means of transport amounts to $2^{16} = 65536$. In case of complex models of the exploitation process of the technical objects, in order to determine the optimal control strategy it is essential to use the proper and effective methods and mathematical tools. At work, as the tool for choosing the optimal strategy δ^* of controlling the availability of the means of transport, on the basis of the semi-Markov model of process exploitation the following genetic algorithm was implemented [4, 12]. General scheme of choice of the optimal strategy using genetic algorithm is presented in Figure 2.

In case of the implementation of the genetic algorithm to determine the optima strategy of controlling the exploitation processes for technical objects (e.g. controlling availability of the means of transport), on the basis of the semi-Markov model of the exploitation process the following guidelines should be considered:

- the examined stochastic process is the *m*-state decisive semi-Markov process,
- strategies (stationary and determining ones) are the functions converting the set of the states of the process into the set of decisions, possible for implementation in each of these states,
- in each state it is possible to implement one of the two ways of performance (called a decision),

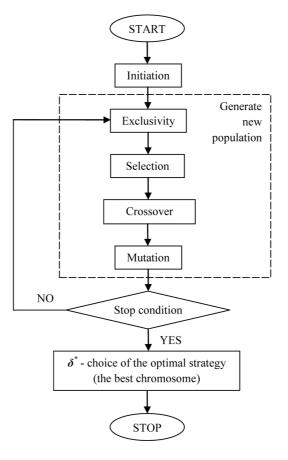


Fig. 2. General scheme of the genetic algorithm for determining the optimal strategy δ^*

 if the decisions are marked as 0 and 1 then the set of stationary and determining strategies will be the set of functions:

$$\delta: S \to D,$$

where:

S – is the set of the states of the process, $S = \{1, 2, ..., m\}$,

D – is the set of decisions made in the states of the process, $D = \{0,1\}$.

On the basis of the following guidelines each possible control strategy can be presented as m-positioning sequence consisting of 0 and 1. This is then the positioning binary number. Therefore, an exemplary control strategy for the model of the exploitation process consisting of 16 states can be determined in the following way:

 $\delta = [1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1].$

5. Summary

The results of the research presented in the article constitute a partial solutions from the works implemented within the broader project which aimed at preparing a complex control method for the exploitation process of the means of transport with the usage of the decisive semi-Markov processes.

The presented method controlling availability of the technical objects (means of transport) means determination a proper strategy (a sequence of control decisions made in individual states of the model process) for which the function constituting the criterion of evaluation achieves an extreme value. In order to specify the optimal strategy controlling the availability of the technical objects the genetic algorithm was recommended.

Due to the general character the presented method canoe implemented for solving a broad spectrum of optimization issues concerning the exploitation systems for the technical objects such as: controlling availability and reliability, analysis of costs and profits, analysis of risk and safety etc. In each case there is a necessity to form properly the definition of the criterion and specifying possible control decisions made in the states of the examined exploitation process of the technical objects and estimating entrance date for the mathematical model of the process which consists of the values of the elements included in the matrix of the core of the process Q(t), the matrixes of probabilities for changes P and the conditional matrixes for duration times of the states of the process Θ .

In the following stages of the implemented works, in order to specify the optimal strategy of the control of the exploitation process, implemented in the examined real exploitation system of the means of transport the entrance data for the model of the process and the computer program with the genetic algorithm will be prepared.

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