HEURISTIC ALGORITHMS APPLIED TO THE PROBLEMS OF SERVICING ACTORS IN SUPPLY CHAINS

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Abstract: *The paper discusses main decision problems analysed in the subject matter of servicing actors operating in the supply chains, i.e. the vehicle routing problem, vehicles-to-task assignment problem and the problem of entities' localization in the supply chain. The input data used to describe supply chains is given as well as the basic constraints and the criterion functions used in the development of mathematical models describing the supply chains. Servicing actors in supply chains is the complex decision making problem. Operators in the supply chains are constrained by: production capacity of the suppliers, the demand of the customers in particular working days, storage capacities of warehouses, handling capacities of warehouses, suppliers' and warehouses' time windows and other. The efficiency of supply chain is described by cost of transport between operators, costs of passing cargoes through warehouses and delivery time to the recipient. The heuristic algorithms, like genetic and ant algorithms are detailed and used to identify issues related to the operation of actors operating in the supply chains are described. These algorithms are used for solving localization problems in supply chains, vehicle routing problems, and assignment problems. The complexity of presented issues (TSP is known as NP-hard problem) limits the use of precise algorithms and implies the need to use heuristic algorithms. It should be noted that solutions generated by these algorithms for complex decision instances are sub-optimal solutions, but nonetheless it is accepted from the practical point of view.*

Key words: *supply chain, genetic algorithm, ant algorithm.*

1. Introduction

Under the high competition in the logistics market, developing strategies for servicing actors involved in the tasks execution along the supply chains becomes a crucial thing. Servicing of actors placed in a given logistic network is defined through delivery deadlines, delivery readiness, quality and flexibility of deliveries, timely completion of transport, complete order execution, receipt and shipment of goods without any damage (Nowakowski, 2004). Evaluation of the quality of service of entities operating in the supply chain is also related to the reliability of these services. Reliability of deliveries (Jacyna-Gołda and Lewczuk, 2017; Twaróg, 2005) is the quotient of the timely performed orders to the total number of orders. The quality of supplies can be measured by the quotient of the number of claims to the total

number of orders. Flexibility of supplies (ability to adapt to customer needs) can be specified by the number of special wishes fulfilled to the total number of requests of this type. The availability to deliver supplies can be expressed by the quotient of the number of orders completed to the total number of orders.

In the nowadays economy, logistics services buyers increasingly submit sophisticated expectations as to the quality of the products. Compliance with the requirements is not easy, especially as long as the transport of goods from manufacturers to the customers is attended by many participants: operators, forwarders, carriers, etc. In addition, the process of cargo flow is no longer subjected only to price, but the quality earns important place. One of the elements of service quality is the time and efficiency of its implementation. Taking into

account the above it can be stated that modern supply chains should characterize by (Jacyna-Gołda, et al., 2013; Jacyna-Gołda et al., 2016):
- immediate response capacity and ability to meet rapidly changing demand,
- flexibility, that is the ability to adapt to market needs,
- ability to make optimal use of company resources,
- ability to use all available information.

In order to improve material flow between elements of the logistics network which determine the time of tasks realization, new methods and algorithms solving key decision problems are developed for:
- location of the facilities in logistics network, i.e. warehouses,
- the choice of shortest routes, i.e. TSP,
- allocation of vehicles to transport tasks.

The main factor determining the quality of services is time of order fulfilment. Delivery time is determined by the distance between particular facilities in the logistics network. The priority is to set this location (Jacyna-Gołda et al., 2016; Wasiak et al., 2016). Handling bodies operating in the supply chain requires designation of minimum routes for the vehicles carrying specific transport task. The search for the shortest paths is still very up to date and refers to the classic traveling salesman problem. Many researchers dealing with this problem are looking for efficient algorithms for finding the shortest paths (Izdebski and Jacyna, 2012 , 2013; Izdebski, 2014; Jacyna-Gołda et al., 2016, 2017; Szczepański and Jachimowski, 2014; Szczepański et al., 2014; Wasiak et al., 2017; Zieja et al., 2015). In turn, the assignment of vehicles to the task is considered to minimize the number of vehicles while setting the minimum routes for all transport tasks.

Heuristic algorithms: genetic and ant algorithm are presented in the paper, and described as a tools supporting handling bodies in supply chain. These algorithms are used for solving localization problems in supply chains, vehicle routing problems, and assignment problems. The complexity of presented issues (TSP is known as NP-hard problem) limits the use of precise algorithms and implies the need to use heuristic algorithms. It should be noted that solutions generated by these algorithms for complex decision instances are sub-optimal solutions, but nonetheless it is accepted from the practical point of view.

2. Supply chains operators

Supply chain is defined as a group of companies such as mining, manufacturing or distribution companies performing common activities to meet the demand for specific products from the acquisition of raw materials to delivery to the final customers (Fig. 1). These activities include development, production, sales, service, supply, distribution, resource management, support activities and other. The role and positioning of individual companies - operators in the supply chain structure results from the division of duties into subsequent stages of production and sales of products. The supply chain can be defined as a network of interdependent associations of organizations, acting on the basis of mutual cooperation, jointly controlling and streamlining material flows and information from suppliers to final customers (Fechner, 2007; Jacyna-Gołda, 2015a, 2015b). It can therefore be said that the actors in the supply chain, through their connections with suppliers and customers, are involved in the various processes and activities that create value in the form of products and services delivered to the final consumer. Each of the integrated operators in the supply chain is responsible for a portion of the flows. These entities can be parts manufacturers of parts, components or final products, logistics services providers as well as customers.
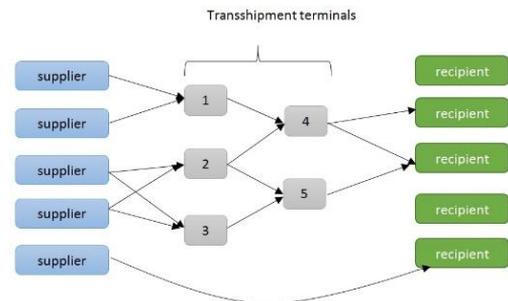

Fig. 1. Operators in supply chain. Source: own work

In order to characterize supply chains, specified input data must be defined:
- $\mathbf{V} = \{ v : v = 1,2,...,v',...,V \}$ – set of nodal elements of logistics network, i.e. suppliers, warehouses, producers etc.,

- $T = \{ t : t = 1,2,...,t',...,T \}$ – set of working days in which the cargoes are delivered to warehouses, receivers or other companies,
- $DS = \{ v : \alpha(v) = 0 \text{ for } v \in \mathbf{V} \}$ – set of suppliers,
- $MS = \{ v : \alpha(v) = 1 \text{ for } v \in \mathbf{V} \}$ – set of warehouses,
- $P = \{ v : \alpha(v) = 2 \text{ for } v \in \mathbf{V} \}$ – set of receivers of cargoes like production facilities,
- $\mathbf{D}1 = [d1(v,v') : d1(v,v') \in R^+, v \in DS, v' \in MS]$ – distance matrix in relations: suppliers-warehouses,
- $\mathbf{D}2 = [d2(v,v') : d2(v,v') \in R^+, v \in DS, v' \in P]$ – distance matrix in relations: suppliers - receivers,
- $\mathbf{D}3 = [d1(v,v') : d3(v,v') \in R^+, v \in MS, v' \in P]$ – distance matrix in relations: warehouses - receivers,
- $\mathbf{Q}1 = [q1(v)]$ – vector of supply volumes,
- $\mathbf{Q}2 = [q2(v,t)]$ – vector of demand from receivers' in particular work-days expressed in palletised loading units,
- $\mathbf{Q}3 = [q3(v)]$ – vector of total demand from receivers' in palletised loading units,
- $POJ = [poj(v)]$ – vector of warehouses storage capacities,
- $\mathbf{K} = [k(v)]$ – vector of unit costs of passing through warehouses,
- $\mathbf{C} = [c(v,v')]$ – vector of unit transport costs between elements of supply chain.

Operators in the supply chains are constrained by: production capacity of the suppliers, the demand of the customers in particular working days, storage capacities of warehouses, handling capacities of warehouses, suppliers' and warehouses' time windows and other. The efficiency of supply chain is described by cost of transport between operators, costs of passing cargoes through warehouses and delivery time to the recipient.

Heuristic algorithms are used to determine the minimum linkage between the supply chain elements. This way of linkage minimizes the cost of transport, time of goods delivery to the customers, and the number of vehicles used. The location of the supply chain elements is determined by the volume of material flow entering the particular operators, so the heuristic algorithm is also responsible for determining the optimal volumes of the flow.

3. Decision problems in supply chains

Handling bodies operating in the supply chain involves the selection of a routes for goods flows from the suppliers to the customers through indirect elements like warehouses or transshipment terminals. The problem of determining driving routes (Abdoun and Abouchabaka, 2011; Bräysy and Gendreau, 2005) (vehicle scheduling problem) refers to the broadly discussed issue of the traveling salesman problem. The TSP problem is an optimization task of finding the Hamilton minimum cycle in a full weighted graph.

The mathematic formulation of travelling salesman problem can be as follows:

- locations visited by salesman are represented by set $\mathbf{M} = \{1,...,i,...,j...M\}$,
- distances between locations to be visited by salesman are defined as a matrix $D = [d(i,j)]$,
- binary decision variable sets the connection between locations taking the 1-value when connection is fixed, and 0-value otherwise: $X = [x(i,j)]$.

Traveling salesman visits each location only once:

$$\forall j \in M \quad \sum_{i=1}^{M} x(i,j) = 1 \tag{1}$$

$$\forall i \in M \quad \sum_{j=1}^{M} x(i,j) = 1 \tag{2}$$

Criterion function sets the minimum-length route for salesman consisting of all visited locations, given as follows:

$$F(x) = \sum_{i=1}^{M} \sum_{j=1}^{M} x(i,j) \cdot d(i,j) \longrightarrow \min \tag{3}$$

An exemplary interpretation of the problem of determining the route for vehicles collecting wastes from individual inhabitants to the landfill is shown in Figure 2. The decision problem in the supply chain defined here is to determine the minimum route for the vehicles reaching waste generating sites. In the presented example, in addition to the classic assumptions of the salesman's problem, the limitations of vehicle's capacity are taken into account, which further complicates the designation of waste collection routes.

Fig. 2. The problem of determining driving routes in municipal enterprises (Izdebski, 2014)

Servicing companies in the supply chains requires assigning vehicles to ongoing tasks in the supply chain. In the classical assignment problem, performers are characterized by certain skills and traits that allow them to perform particular tasks. The solution to the problem is the identification of appropriate contractors for the tasks, so that the contractors will assign those tasks in the optimal way (maximum profit, minimum loss). The allocation problem can be modified by taking into account different combinations of data like number of vehicles equal to number of assigned tasks, number of tasks smaller or larger than number of vehicles. The mathematical formulation of the classical assignment problem is identical to that of the traveling salesman.

To accomplish all tasks with a minimum number of vehicles, the minimal number of paths covering all nodes of the network must be find out first according to classic assignment problem version (Fig. 3a). The problem is reduced to the preparing case-related bipartite graph and determining the maximal association in this graph. Fig. 3b presents graph where the vertices are the starting and ending points of the transport task. In addition, there are time limits in the supply chain and limited ability to perform multiple tasks with one vehicle.

The location of the entities operating in the supply chains is crucial for its quality. The problem of locating supply chain elements like warehouses, suppliers or customers is widely discussed as important decision problem (Ambroziak et al., 2006; Brandeau and Chiu, 1989). The classic location problem is described in the literature as Capacitated Warehouse Location Problem CWLP (Akinc and Khumawala, 1977; Sharma and Berry, 2007). The cost of transport depends on the quantity of goods transported between warehouses and customers, and the distance between them, so the quantity of goods to be transported must also be determined. In case where the capacitive restriction is not taken into account, the problem turns into Uncapacitated Warehouse Location Problem (Khumawala, 1973).

Depending on the complexity of supply chain in which the entities are localized, classic version of the problem is modified and expanded with additional constraints. An exemplary function that determines the cost of transport between suppliers, storage facilities and customers can be presented as:

$$f(\mathbf{Y}, \mathbf{Z}) = \sum_{r \in R} \left(\sum_{i \in I} \sum_{m \in M} c_{im}^r \cdot y_{im}^r + \sum_{m \in M} \sum_{j \in J} k_{mj}^r \cdot z_{mj}^r \right) \quad (4)$$
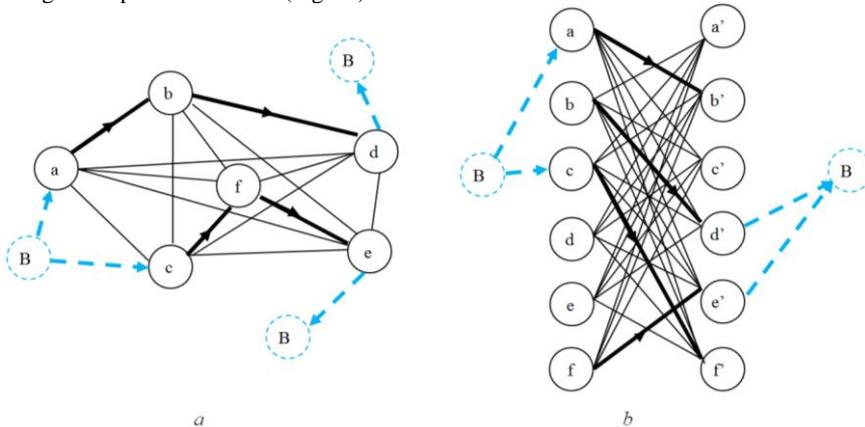

Fig. 3. Graphical interpretation of assignment, a) network, b) graph (Burkrd et al., 2009)

where:

c_{im}^{r} – unit costs of delivery of goods of *r*-th commodity group from *i*-th suppliers to *m*-th warehouses;

y_{im}^{r} – volume of supplies of *r*-th commodity group from *i*-th suppliers to *m*-th warehouses;

k_{mj}^{r} – unit costs of delivery of goods of *r*-th commodity group from *m*-th warehouses to *j*-th receivers;

z_{mj}^{r} – volume of supplies of *r*-th commodity group from *m*-th warehouses to *j*-th receivers;

$\mathbf{R} = \{1,\ 2,\ ...,\ r,\ ...,\ R\}$ – set of commodity groups.

Complexity of discussed decision problems forces using heuristic algorithms. These problems are essentially based on the minimization of vehicle routes in a given supply chain, and this in turn restricts the use of precise algorithms in these types of problems.

4. Heuristic algorithms to plan servicing operators in supply chains

The heuristic algorithms used in solving complex decision problems in the aspect of servicing entities operating in the supply chain are genetic and ant algorithms. Both algorithms are well recognized and investigated.

To describe the components of the ant algorithm a set ***WTP*** of route points visited by a single ant is defined. With reference to the supply chain requirements and decision problems like determining rational driving routes or locations, the route points may be suppliers of raw materials for production companies (Jacyna-Gołda and Izdebski, 2017) or households generating wastes, placed in the logistics network of municipal enterprises (Izdebski, 2014). The vehicles-to-tasks assignment problems define route points as transport tasks to which vehicles are assigned (Jacyna and Izdebski, 2014).

$$\mathbf{WTP} = \{1,...,y,...,z,...,\overline{\mathrm{WTP}}\} \qquad (5)$$

where $\overline{\mathrm{WTP}}$ - cardinality of set $\mathbf{WTP}$, $y \neq z$.

The starting point of each ant's route is a randomly chosen route point, eg supplier. Further travel of ant, and thus selection of consecutive points, occurs with a certain probability (Dorigo and Stutzle, 2004):

$$\mathrm{PR}^{mr}_{yz}(t) = \begin{cases} \dfrac{\left[\tau_{yz}(t)\right]^{\alpha} \cdot \left[\eta_{yz}(t)\right]^{\beta}}{\sum_{l \in \Omega^{mr}} \left[\tau_{yl}(t)\right]^{\alpha} \cdot \left[\eta_{yl}(t)\right]^{\beta}} & , z \in \Omega^{mr} \\ 0 & , z \notin \Omega^{mr} \end{cases} \qquad (6)$$

where:

$\tau_{yz}(t)$ – intensity of pheromone trace between y -th and z -th route point in t -th iteration,

$\eta_{yz}(t)$ – heuristic data, i.e. $\eta_{yz}(t) = \dfrac{1}{w(y,z)}$, where

w(y,z) - distance between y -th starting point and z -th starting point,

α, β – effect of pheromones and heuristic data on the behavior of ants,

Ω^{mr} – set of so far unvisited l -th starting points by mr -th ant.

A random selection of paths between point *y* and *l*-th points begins with the calculation of the probability of passing to points not yet visited by the ant according to the pattern (6). The next step is to calculate the distribution for each transition path and draw the number *r* from the range [0,1]. A *tr* route with a distribution value q_{tr} is selected if $q_{tr-1} < r \leq q_{tr}$, where *tr* is is the number of route between *y*-th point and the *l*-th points of shipment.

When all ants have completed the routes, the pheromone traces are updated. The process uses a cyclic update (Dorigo and Stutzle, 2004) as the most efficient when compared to the density or quantitative updates. In the beginning it is assumed that the trace on the links between the points is equally strong. In subsequent iterations, the pheromone trace is calculated according to the formula:

$$\tau_{yz}(t+1) = (1-\rho)\tau_{yz}(t) + \sum_{mr=1}^{\mathbf{MR}} \Delta\tau_{yz}^{mr}(t) \qquad (7)$$

where:

mr – another ant in the anthill $mr \in \mathbf{MR}$,

ρ – pheromone evaporation factor $(0 < \rho \leq 1)$,

$\tau_{yz}(t+1)$ – pheromone boost, for the first iteration it takes a value τ_0 on each link.

The first component of formula (7) defines the pheromone evaporation rate, while the second one determines the pheromone boost and takes the values (Dorigo and Stutzle, 2004):

$$\Delta\tau_{yz}{}^{mr}(t) = \begin{cases} \dfrac{1}{L^{mr}(t)} & \text{if route } (y,z) \text{ was} \\ & \text{used by ant } mr \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

where:

$L^{mr}(t)$ – length of route done in t-th iteration by mr-th ant, if route (y,z) was performed by mr-th ant then $\Delta\tau_{yz}{}^{mr}(t)$ is equal to $\dfrac{1}{L^{mr}(t)}$, and 0 otherwise.

The genetic algorithm used for the problems of servicing entities operating in the supply chains has been applied to determine the routes of vehicles, i.e. for the collection of municipal waste from individual households (Izdebski, 2014) or the problems of objects location in the supply chains (Jacyna-Gołda et al., 2016, Szczepański et al., 2014).

Genetic algorithms are based on mechanisms of natural selection and heredity. They are used as a practical optimization tool. The main advantages of genetic algorithms, that dominate other optimization methods, are the search for an optimal point not from a single location in the search plane, but from several points determined by a given population and based on the information determined by the objective function, rather than the derivative. Value-based targeting is a valuable advantage of genetic algorithms. The criteria function provides values that allow the genetic algorithm to find a solution that is acceptable and satisfactory from the point of view of the problem.

The main stages of the algorithm are: selection, crossing and mutation. The selection involves selecting the best individuals from the initial population and including them to the next generation. Various methods of selection are described in the literature (Michalewicz, 1996) i.e. stochastic method with repetitions (roulette method), deterministic sampling method, stochastic method with residues with repetitions, stochastic method with no repeat repetition, tournament method, ranking method. The quality of the solution

(faster convergence to the wanted values) depends on the method used. Crossing involves the exchange of genetic material between individuals of the parental population, resulting in the individuals of the progeny. Depending on the processed case, the crossing takes different forms. The classic genetic algorithm is based on chromosomes. Chromosomes are randomly generated binary (in most cases) sequences creating a whole population of individuals. The classic method of single point crossing is used for such a structure. Other representations of the structure of the chromosome are also possible, like as a sequence of natural numbers. Two-point crossings, such as PMX (partial matched crossover) and OX (order crossover) are introduced into this structure. Mutation involves randomization of genes, conversion of values, or transposition. Generation of the random population is randomized or through other heuristic algorithms. The condition of the end of the algorithm can be: the number of expected iterations or lack of diversity of the population.

Depending on the problem, the structure of the algorithm takes different forms – like in case of assignment of vehicles to tasks in supply chain and determining the route for vehicles. The chromosome takes the form of a sequence of integers interpreted as tasks or suppliers (Fig. 4). In order to determine the location of the objects, the matrix structure was used (Fig. 5).

Fig. 4. Structure of numerical strings (Izdebski and Jacyna, 2013).

	DS1	DS2	MS1	MS2	MS3	P6	P7
DS1	0	0	7	5	10	10	5
DS2	0	0	5	3	15	5	5
MS1	0	0	0	3	3	5	5
MS2	0	0	3	0	2	2	5
MS3	0	0	1	1	0	8	20
P6	0	0	0	0	0	0	0
P7	0	0	0	0	0	0	0

Fig. 5. Matrix structure (Jacyna-Gołda et al., 2016)

The crossing process is adequate to the assumed structure of the chromosome. In the problem of locating objects in the supply chain, two matrices are created: the **DIV** containing the rounded average

values from both parents, and the **REM** matrix with information on whether rounding is needed. Assuming that the values of matrix **M1** and **M2** (parents) in each cell assume the marking $m^1_{v,v'}$, $m^2_{v,v'}$ the values of matrix elements **DIV** and **REM** are calculated from the following relationships:

$$dim_{v,v'} = \left\lfloor (m^1_{v,v'} + m^2_{v,v'}) / 2 \right\rfloor \qquad (9)$$

$$rem_{v,v'} = (m^1_{v,v'} + m^2_{v,v'}) / \mod 2 \qquad (10)$$

Based on the matrix **DIV** and **REM** new matrix structures are defined. The **REM** matrix contains even number of ones. Taking pairs of generated ones and adding them to the values from the **DIV** matrix

two distinct matrix structures are created. A graphical representation of the crossing process is shown in Fig. 6, Fig. 7 and Fig. 8 (*h*-th cargo shipped in the supply chain).

The principle of the mutation operator is to draw two numbers *p* and *q* from the range $2 \le p \le k$ and $2 \le q \le n$, which determine the number of rows and columns of submatrix of dimensions k × n (*k* - number of rows in the main matrix (processed by the algorithm), *p* - number of columns in the main matrix), I in this part. Generated matrix is modified so that the sum of values in the columns and rows before and after the modification was not changed. The mutation process is shown in Fig. 9. The sum before and after in the generated sub-matrix is the same.

M1(*h*)

	D1	D2	TZ1	TZ2	TZ3	TZ4	O1	O2
D1	0	0	20	10	0	0	10	15
D2	0	0	15	20	0	0	5	5
TZ1	0	0	0	15	10	10	0	0
TZ2	0	0	0	0	20	25	0	0
TZ3	0	0	0	0	0	10	10	0
TZ4	0	0	0	0	0	0	10	35

M2(*h*)

	D1	D2	TZ1	TZ2	TZ3	TZ4	O1	O2
D1	0	0	20	10	0	0	7	18
D2	0	0	15	20	0	0	8	2
TZ1	0	0	0	15	10	10	0	0
TZ2	0	0	0	0	20	25	0	0
TZ3	0	0	0	0	0	10	10	0
TZ4	0	0	0	0	0	0	10	35

Fig. 6. Structures designed for the crossing (Jacyna-Gołda et al., 2016)

DIV(*h*)

	D1	D2	TZ1	TZ2	TZ3	TZ4	O1	O2
D1	0	0	20	10	0	0	8	16
D2	0	0	15	20	0	0	6	3
TZ1	0	0	0	15	10	10	0	0
TZ2	0	0	0	0	20	25	0	0
TZ3	0	0	0	0	0	10	10	0
TZ4	0	0	0	0	0	0	10	35

REM(*h*)

	D1	D2	TZ1	TZ2	TZ3	TZ4	O1	O2
D1	0	0	0	0	0	0	1	1
D2	0	0	0	0	0	0	1	1
TZ1	0	0	0	0	0	0	0	0
TZ2	0	0	0	0	0	0	0	0
TZ3	0	0	0	0	0	0	0	0
TZ4	0	0	0	0	0	0	0	0

Fig. 7. Aiding matrices in the crossing process (Jacyna-Gołda et al., 2016)

	D1	D2	TZ1	TZ2	TZ3	TZ4	O1	O2
D1	0	0	20	10	0	0	9	16
D2	0	0	15	20	0	0	6	4
TZ1	0	0	0	15	10	10	0	0
TZ2	0	0	0	0	20	25	0	0
TZ3	0	0	0	0	0	10	10	0
TZ4	0	0	0	0	0	0	10	35

	D1	D2	TZ1	TZ2	TZ3	TZ4	O1	O2
D1	0	0	20	10	0	0	8	17
D2	0	0	15	20	0	0	7	3
TZ1	0	0	0	15	10	10	0	0
TZ2	0	0	0	0	20	25	0	0
TZ3	0	0	0	0	0	10	10	0
TZ4	0	0	0	0	0	0	10	35

Fig. 8. New matrix structures (Jacyna-Gołda et al., 2016)

	D1	D2	TZ1	TZ2	TZ3	TZ4	O1	O2
D1	0	0	20	10	0	0	10	15
D2	0	0	15	20	0	0	5	5
TZ1	0	0	0	15	10	10	0	0
TZ2	0	0	0	0	20	25	0	0
TZ3	0	0	0	0	0	10	10	0
TZ4	0	0	0	0	0	0	10	35

	D1	D2	TZ1	TZ2	TZ3	TZ4	O1	O2
D1	0	0	20	10	0	0	7	18
D2	0	0	15	20	0	0	8	2
TZ1	0	0	0	15	10	10	0	0
TZ2	0	0	0	0	20	25	0	0
TZ3	0	0	0	0	0	10	10	0
TZ4	0	0	0	0	0	0	10	35

Fig. 9. Process of matrix mutation (Jacyna-Gołda et al., 2016)

The partial matched crossover (PMX) is a practical solution in the problem of defining routes for vehicles serving supply chain entities and in the problem of allocating these vehicles to tasks. This crossing ensures that the chromosome is correct and has acceptable construction. With such a crossing, the dual points of start in the route are not the case, because the vehicle reaches each point only once (according to the constrains of travelling salesman problem). PMX crossing is one of the types of crosses used in permutation representation tasks. The principle of crossing is shown in Fig. 10. The crossing is based on the random selection of two chromosomes for crossing. The crossing parameter is the probability determining how many individuals will cross. It is determined at the beginning of the algorithm. When chromosomes are designated to cross, they are randomly combined into pairs. If the number of selected chromosomes is odd, randomly selected chromosome from the population is added to complete the crossing set. The mutation process involves the random exchange of genes in a numerical sequence.

5. Conclusions

Servicing the supply chain operators is a complex decision-making task that requires addressing the issues of routing, the allocation of vehicles to tasks, and addressing the problem of locating objects in the supply chains. All these things determine the time spent by the operators, which is a key factor in assessing the efficiency of the supply chain. The computational complexity of presented issues implies the choice of heuristic methods to solve them. It should be noted that the quality of the generated results by the heuristic algorithms depends on the setting parameters of these algorithms, such as the crossing and mutation parameters, population size, number of iterations and other. The parameter selection should be done in experimental way.

References
[1] ABDOUN, O., ABOUCHABAKA, J., 2011. A Comparative Study of Adaptive Crossover Operators for Genetic Algorithms to Resolve the Traveling Salesman Problem. *International Journal of Computer Applications, Foundation of Computer Science*, 31 (11), pp. 49-57,
[2] AKINC, U., KHUMAWALA, B.M., 1977. An efficient branch and bound algorithm for the capacitaded warehouse location problem. *Management Science*, 23 (6), pp. 585–594.
[3] AMBROZIAK, T., JACYNA, M., WASIAK, M., 2006. The Logistic Services in a Hierarchical Distribution System, in: Goulias, K.: *Transport Science and Technology*, ELSEVIER, pp. 383-393.
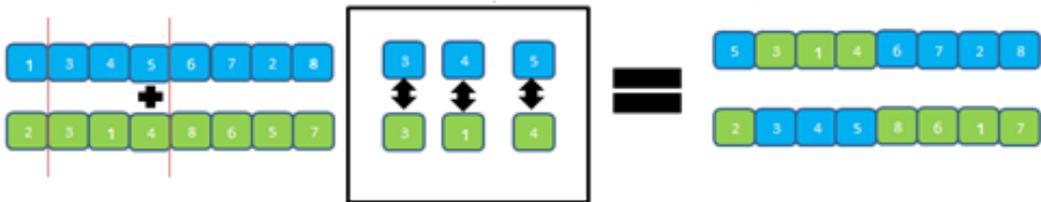

Fig. 10. PMX crossing (own work on the base of Michalewicz, 1996)

[4] BRANDEAU, M. L, CHIU, S.S., 1989. An Overview of Representative Problems in Location Research. *Management Science*, 35 (6), pp. 645–674.

[5] BRÄYSY, O., GENDREAU, M., 2005. Vehicle Routing Problem with Time Windows, Part II: Metaheuristics. *Transportation Science*, 39 (1), pp. 119-139.

[6] BURKARD, R., DELL'AMICO, M., MARTTELO, S., 2009. *Assignment problems.* Philadelphia: Society for Industrial and Applied Mathematics..

[7] DORIGO, M., STUTZLE, T., 2004. *Ant Colony Optimization*. USA: Bradford Books.

[8] FECHNER, I., (ed.), 2007. *Zarządzanie łańcuchem dostaw*. Poznan: Wyższa Szkoła Logistyki.

[9] IZDEBSKI, M., 2014. The use of heuristic algorithms to optimize the transport issues on the example of municipal services companies. *Archives of Transpor*t, 29 (1), pp. 27-36.

[10] IZDEBSKI, M., JACYNA, M., 2012. Model przydziału zasobów do zadań w przedsiębiorstwie transportowym. *Logistyka* 4/2012, pp. 247-254.

[11] IZDEBSKI, M., JACYNA, M., 2013. Wybrane aspekty zastosowania algorytmu genetycznego do rozwiązywania problemu przydziały zasobów do zadań w przedsiębiorstwie transportowy. *Prace Naukowe Politechniki Warszawskiej. Transport*, 97, pp. 183-194.

[12] JACYNA, M., IZDEBSKI, M., 2014. The ant algorithm for solving the assignment of vehicles to tasks in the municipal services companies. *Journal of Kone*s, 21 (2), pp.113-119.

[13] JACYNA-GOŁDA, I., 2015a. Evaluation of operational reliability of the supply chain in terms of the control and management of logistics processes, in NOWAKOWSKI, T. (ed.) *Safety and Reliability: Methodology and Applications*, CRC Press Taylor & Francis Group, pp. 549-558.

[14] JACYNA-GOŁDA, I., 2015b. Chosen problems of effective supply chain management of companies. *Journal of KONES*, 22 (2), pp. 89-96.

[15] JACYNA-GOŁDA, I., GOŁĘBIOWSKI, P., IZDEBSKI, M., KŁODAWSKI, M., JACHIMOWSKI, R., SZCZEPAŃSKI, E., 2017. The evaluation of the sustainable transport system development with the scenario analyses procedure. *Journal of Vibroengineering*, 19 (7), pp. 5627-5638.

[16] JACYNA-GOŁDA, I., IZDEBSKI M., PODVIEZKO A., 2016. Assessment of the efficiency of assignment of vehicles to tasks in supply chains: A case-study of a municipul company. *Transport*, 31 (4), pp. 1-9.

[17] JACYNA-GOŁDA, I., IZDEBSKI, M., 2017. The Warehouse Location Problem in the Context of Vehicle Routing Problem in the Production Companies, in Macioszek, E., Sierpiński, G. (ed.), *Recent Advances in Traffic Engineering for Transport Networks and Systems*, Springer, pp 50-62.

[18] JACYNA-GOŁDA, I., IZDEBSKI, M., SZCZEPAŃSKI, E., 2016. Assessment of the Method Effectiveness for Choosing the Location of Warehouses in the Supply Network, *Challenge of Transport Telematics*, Springer vol. 640, pp.84-97.

[19] JACYNA-GOŁDA, I., LEWCZUK, K., 2017. The method of estimating dependability of supply chain elements on the base of technical and organizational redundancy of process. *Eksploatacja i Niezawodnosc – Maintenance and Reliability*, 19 (3), pp.382–392.

[20] JACYNA-GOŁDA, I., SANTAREK, K., WASIAK, M., 2013. Location of warehouse facilities in the logistics network, in: FELIKS, J. (ed.) *CLC 2013: Carpathian Logistics Congress - Congress Proceedings*.

[21] KHUMAWALA, B. M., 1973. An efficient heuristic procedure for the uncapacitated warehouse location problem, *Naval Research Logistics Quarterly*, 20 (1), p. 109–121.

[22] MICHALEWICZ, Z., 1996. *Algorytmy genetyczne + struktury danych = programy ewolucyjne*, Warsaw: Wydawnictwo Naukowo – Techniczne.

[23] NOWAKOWSKI, T., 2004. Reliability model of combined transportation system, in SPITZER, C. et al. *Probabilistic Safety Assessment and Management*. London: Springer.

[24] SHARMA, R. R. K., BERRY, V., 2007. Developing new formulations and relaxations of single stage capacitated warehouse location problem (SSCWLP): Empirical investigation

for assessing relative strengths and computational effort. *European Journal of Operational Research*, 177 (2), pp. 803–812.

[25] SZCZEPAŃSKI, E., JACYNA-GOŁDA, I., MURAWSKI, J., 2014. Genetic algorithms based approach for transhipment HUB location in urban areas. *Archives of Transport*, 31 (3), pp.73-78.

[26] SZCZEPAŃSKI, E., JACHIMOWSKI, R., 2014. Simulation analysis of vehicle routing problem solution, in: FELIKS, J. (ed.) *CLC 2013: Carpathian Logistics Congress - Congress Proceedings*, Ostrava: TANGER Ltd., pp. 446-452

[27] TWARÓG, J., 2005. *Mierniki i wskaźniki logistyczn*e. Poznan: Biblioteka Logistyka.

[28] WASIAK, M., JACYNA, M., LEWCZUK, K., SZCZEPAŃSKI, E., 2017. The method for evaluation of efficiency of the concpept of centrally managed distribution in cities. *Transport*, 32 (4).

[29] WASIAK, M., JACYNA-GOŁDA, I., IZDEBSKI, M., 2016. Multi-criteria warehouses location problem in the logistics network. *ICIL 2016 Conference Proceedings,* pp. 352-364.

[30] ZIEJA, M, SMOLOŃSKI, H., GOŁDA, P., 2015. Information systems as a tool for supporting the management of aircraft flight safety. *Archives of Transport*, 36 (4), pp. 67-76.