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Formal Notation of a Logistic System Model Taking into Consideration Cargo Stream Transformations

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Abstract

The modeling of logistic systems is a complex task. Specific goals to be attained by the systems (mainly cargo stream form transformations) and the diversity of resources used within the systems are the main reasons. Therefore various models useful for the partial analyses of logistic systems have been devised. The author's approach to the formal notation of a logistic system model proposed in this paper may be used as a basis to work out various classes of detailed (integrated) models. In addition to cargo stream time, form and location transformations, transformations of material and human resources used within logistic systems have also been taken into account in the proposed model. **Keywords:** logistic systems, system potential, logistic system modeling

1. Introduction

Various definitions of the notion of logistic system may be found in literature. However, they do not contradict each other, but rather emphasize various logistics aspects. Definitions derived from the general definition of a system take into consideration, first of all, the structure of the system, and specify such systems as sets of elements mutually inter-related by respective relations. Among such definitions there is the one given by P. Blaik [3, p. 48; 5, p. 53] according to which a logistic system is "a set of logistic elements in which mutual relations are specified by means of some transformations". Hence logistic systems are the system result from the function of space and time, quantitative and qualitative transformation processes

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accomplished during the flow of goods and information (cf. [4, p. 53]). Similarly, in opinion of G. Ghiani, a logistic system is made up of a set of facilities linked by transportation services. These facilities are sites where streams of materials are processed, e.g. manufactured, stored, sorted, sold or consumed [10, p. 1].

Handling of material goods is inherent to transformations the goods undergo (and transformations of related flows of information). Taking this into consideration, it has been assumed that the purpose of a logistic system is to transform cargo streams and related information streams.

Specific processes occurring in each logistic system are known as logistic processes. In general, a logistic process is a sequence of subsequent changes in the state of a logistic system that links the original state of system with its final state. Changes in the state of a logistic system are related to the cargo stream time, location and form transformations accomplished in that system ([8, p. 168]).

It follows that location, form and time transformations of the streams of goods exist in logistic processes. S. Piasecki describes these three basic kinds of transformations as "shifts", "transformations" and "delays", respectively [20, p. 18-19]. In practice, pure location, form or time transformations do not occur, as cargo/information streams are transformed with regard to two or even three of the above criteria.

Both the logistic systems themselves and the processes realized within such systems are the subject of logistics. According to J. Fijałkowski, "the subject of applied logistics consists in the theory and practice of transforming cargo/information streams with respect to location, form and time, with the aim of accomplishing logistic tasks at the required service level and at the lowest possible costs" [9, p. 10].

Improvement of logistic processes, which are the inherent elements of all economic processes, is more and more often the most significant way to improve the enterprise's competitive edge. Methods to improve logistic processes have been developed for many years with the aim of identifying still undiscovered areas where waste might be reduced. One strives to keep the logistic systems themselves up with the continually evolving market environment, i.e. to adapt them to the emerging capabilities regarding the effectiveness of logistic processes. The main reasons to modernize the existing logistic systems quoted in the literature include [7, p. 558-561]:

- evolving customer requirements in regard to the level and scope of logistic services,
- changes on supply/delivery markets, e.g. due to population migration, expansions in new markets, new supply sources,
- changes in business ownership (mergers, acquisitions, take-overs),
- high costs of logistic operations,
- competition pressure,
- significant changes within the company e.g. diminished scale of operations, expanded range of the manufactured products.

One of the problems is setting the so called logistic system potential. Logistic potential is defined as a measure of the system's capability to function. Depending on the kind of resources used in logistic processes¹, the human potential, technical potential, material potential, and management potential of a logistic system are distinguished [6, p. 75-76]. To simplify the quoted understanding of the notion of logistic system potential, the material potential was omitted in the further discussion and the management potential was to a large extent simplified, assuming that the logistic system potential is a function of:

- the number of employees employed at individual work stations and indicators of their readiness to do their jobs (the human potential),
- the number of fixed assets of individual categories and indicators of possibility to use them (the technical potential),
- the number of work shifts and deployed organizational principles for cargo/information stream flows (the management potential).

Various methods are used to find out how big the potential of a logistic system is (e.g. [1, p. 194-203], [2, p. 197-213], [5, p. 144-151], [8, pp. 269], [14, p. 105-117], [15, p. 93-96, 124-125 and 211-213], [17, p. 310-327], [21, p. 802-804], [23, p. 200-210], [24, p. 703-711, 717-728]). They include both classical methods applied in engineering of logistic systems, as well as various optimization/modeling methods.

Due to its objective nature, the field of mathematical modeling has been recognized as the major source of methods developed to improve logistic processes. Therefore, more advanced methods are strived for to improve logistic processes, development of mathematical models that reflect the complexity of the processes and implementation of computer methods for solving them.

In this paper, the author's proposal of the basic logistic system model was proposed, taking into account the specificity of cargo stream transformations together with infrastructure and human resources applied to transform the streams. It has been assumed that the logistic system elements include some specially allotted areas, in which cargo stream form and time transformations take place, whereas location transformations occur while cargo is moved from one area to another. Increased complexity of formal notations of cargo stream transformations is a disadvantage of such an approach. On the other hand, a reduction in computational complexity to solve detailed models built in line with that approach is its clear advantage.

2. Formal Model of a Logistic System

To be able to process cargo/information streams, a logistic system must have a suitable structure characterized by certain features (characteristics), see e.g. [13,

¹ In a wider aspect, due to conditions of market activity of the enterprises their human, technical, information, structural, financial, product and market potential is distinguished [18, p. 53].

p. 111-114] or [29, p. 526-528]. The elements of that structure use some resources that make up the potential of the logistic system. The logistic system structure is determined by the location of some logistic objects, which make up a set of elements of that system, as well as by some functional areas distinguished within the objects. Both the objects and the areas are mutually bound by relevant relations². On the other hand, the characteristics of the logistic system structure depend on the infraand supra-structure installed within the individual functional areas of the system and on the employees hired to run the system.

Taking the above into account, it has been assumed that the logistic system structure may be described by a graph (GS) and defined characteristics (FS) of individual elements of that structure. The characteristics are dependent on the technical and human resources and on the organization solutions used in the elements of the logistic system structure ZO.

The structure described by relevant characteristics consists of passive elements that enable tasks assigned to the system to be accomplished. In logistics, those tasks are referred to as the cargo stream time, location and form transformations and are collectively called the logistic tasks. Thus, the tasks depend on cargo stream categories supported by the given system and on transformations to be accomplished on those streams. Logistic tasks have been formally denoted here as **ZL**.

Since in general the logistic tasks assigned to a logistic system of a given structure and characteristics of its elements dependent on the selection of potential of the logistic system may be accomplished in many ways, it has been assumed that the aim of modeling the logistic systems is to identify their potential at which the best way to realize the given logistic task is obtained. The potential that produces the best values of the adopted optimization estimators (indicators that allow each possible solution to be numerically estimated) while satisfying all of the existing constraints (boundary conditions) of the to-be-accomplished logistic task is sought after. Therefore, the potential should meet the accepted principles of accomplishment of the given task, which in formal notation are clearly expressed by restrictions and the objective function.

The logistic task accomplishment principles applied in various logistic systems may vary, but in each case they may be identified and mapped. The accepted logistic task accomplishment principles have been denoted here as ZR.

Taking the above into account, the logistic system model *MSL* has been defined as:

$$MSL = \langle ZL, ZO, GS, FS, ZR \rangle$$
(1)

where:

ZL – logistic task assigned to the given logistic system,

ZO – types of technical and human resources and organizational solutions that can be used in a given logistic system,

² Relations occurring in the logistic system structure mutually bind in some way the elements of that system, making up the so-called cargo processing technological lines.

- **GS** logistic system structure graph,
- FS set of characteristics of logistic system structure elements,
- ZR set of accepted logistic task accomplishment principles.

The subject of engineering of such systems is to identify the logistic system material resources necessary to efficiently accomplish cargo/information streaming and to ensure that the resources are rationally used. Besides, taking into account the organizational aspects related to the efficient utilization of enterprise resources, it should be stated that one of the design goals is to ensure the optimal potential (or rather optimal set of capabilities) for each designed system³.

The technical equipment of logistic systems necessary to transform cargo streams is described by a general profile (intended use), quantity profile (number of pieces of equipment), and economic-technical profile (price, investment outlays necessary for deployment, unit operational costs, efficiency). The human resources used in logistic systems are described in a similar way.

In the later part of the paper, a proposed scope of mapping the logistic system structures, profiles of its elements, and logistic tasks are described. Mapping of the means of work and workforce is also included. Those items are the most interesting subjects while looking for the optimal potential of a logistic system.

3. Mapping of Cargo Streams within Logistic Systems – a Logistic Task

As it was seen in the previous chapter, each logistic system, regardless of its scale, processes cargo streams and streams of related information; the processing consists in form, time and location transformations of the cargo/information streams. A set of the supported **cargo stream categories** is defined as:

$$R = \{r : r = 1, ..., \bar{R}\}$$

For the needs of this research it has been assumed that each cargo stream transformation is a single-criterion transformation, where the selected criterion reflects the primary goal of the transformation. Such an approach makes it possible to treat e.g. the operation of unloading the external means of transport as a form transformation even if the operation actually involves form, time and location transformations.

A set of **cargo stream transformations** accomplished within a logistic system is defined as:

$$\boldsymbol{P} = \left\{ p : p = 1, ..., \bar{P} \right\}$$

³ Precise explanation of the notions of resource and potential as well as the relation between both notions can be found e.g. in [6, p. 71-78].

Taking into consideration that each cargo stream transformation belongs to a single category, the kp mapping that transforms each element of the P set into 1, 2 or 3 may be defined:

$$kp: \mathbf{P} \rightarrow \{1, 2, 3\}$$

where kp(p) = 1/2/3 if the p^{th} cargo stream transformation is a transformation with regard to form, time and location, respectively.

The *kp* transformation allows the following to be defined:

• **PP** set of cargo stream form transformations:

$$\boldsymbol{PP} = \{ p \in \boldsymbol{P} : kp(p) = 1 \},\$$

• *PT* set of cargo stream time transformations:

$$\boldsymbol{PT} = \{ p \in \boldsymbol{P} : kp(p) = 2 \},\$$

• *PM* set of cargo stream location transformations:

$$\boldsymbol{P}\boldsymbol{M} = \{ p \in \boldsymbol{P} : kp(p) = 3 \}.$$

The cargo stream **form transformations** *PP* accomplished in logistic systems are related to changes in the state of the cargo shipment (e.g. formed into palette units, unloaded from external means of transport etc.) or some value-added operations (e.g. labels put on packages). Therefore, various material streams may be combined or separated during such transformations. For example, some palettes and collective containers may be combined while cargo is formed into palette units, whereas during the unloading some cargo from external trucks may be distributed among empty internal transport vehicles and/or palette units.

Packages, equipment used to form shipment units (e.g. palettes, containers) as well as empty vehicles shall be referred to as the cargo carriers in further discussion. The carriers, e.g. empty containers, equipment used to form shipment units and empty means of external transport, shall be included in the R set of cargo categories (apart from cargo itself). Taking into consideration the above-mentioned assumptions, the following categories of cargo stream form transformations may then be distinguished:

- 1⇒1 same type cargo units are processed using materials not regarded as a cargo (e.g. packaging/foil-wrapping/marking/labeling operations),
- 1⇒2 same type cargo units are split-up, whereby the cargo carriers which have been used so far are freed up (e.g. a truck unloading operation),
- 2⇒1 same type cargo units are combined using a specified cargo carrier (e.g. a truck loading operation),
- 2⇒2 same type cargo units are first split-up and then combined, whereby different cargo carriers are used before and after the operation (e.g. a picking operation or unloading external transport containers onto palettes).

Each of the above transformations concerns a category and quantity of the processed cargo stream and the resulting streams. Therefore, the following was defined for the needs of the research:

- $\mathbf{R}\mathbf{p}^p = \{r \in \mathbf{R} : rp(p,r) = 1\}, p \in \mathbf{PP}$ set of cargo streams transformed in the p^{th} transformation,
- $\mathbf{Rk}^{p} = \{r \in \mathbf{R} : rk(p, r) = 1\}, p \in \mathbf{PP}$ set of cargo streams produced by the p^{th} transformation,
- $Qp^{p,r} \in \mathcal{N}$ for $p \in PP$, $r \in Rp^p$ number of the r^{th} category cargo units subject to the p^{th} form transformation,
- $Qk^{p,r} \in \mathcal{N}$ for $p \in PP$, $r \in Rk^p$ number of the r^{th} category cargo units produced by the p^{th} form transformation.

The Rp^p and Rk^p set elements are ordered pairs for transformations of category $2\Rightarrow 2$. However, for transformations of category $1\Rightarrow 1$, $1\Rightarrow 2$ and $2\Rightarrow 1$ one or two of these sets may contain single elements instead of the ordered pairs.

A set of all characteristics of cargo stream form transformations has been defined as:

$$\boldsymbol{F}_{\boldsymbol{P}} = \{\boldsymbol{R}\boldsymbol{p}^{p}, \boldsymbol{R}\boldsymbol{k}^{p}, \boldsymbol{Q}\boldsymbol{p}^{p,r}, \boldsymbol{Q}\boldsymbol{k}^{p,r} : p \in \boldsymbol{P}\boldsymbol{P}\}$$

Each **time transformation** *PT* represents a purposeful halt in the cargo stream flow. Such halts may be executed to perform a warehousing operation, buffering operation or check-up operation. As a result, maintenance of the cargo form and normal planned exclusion of the cargo load from the processing flow are especially characteristic of the time transformations. Cargo stream categories, subject to a given transformation, may be identified for each time transformation. Similarly, the time the cargo is to be excluded from the flow (the so-called standard storage time) may also be identified⁴. The standard storage time is one of the characteristics of the functional areas of the logistic system in which time transformations are accomplished.

The set of numbers of cargo streams subject to the p^{th} time transformation has been defined as $\mathbf{R}_T^p = \{r \in \mathbf{R} : rt(p, r) = 1\}, p \in \mathbf{PT}$.

The location transformations *PM* consist in purposeful movements of cargo streams between various logistic system areas. Neither the form of the cargo is modified nor planned flow interruptions occur. Hence the categories of the cargo streams in question are the basic characteristics of such transformations. Cargo stream characteristics profoundly influence the selection of equipment and employees assigned to accomplish location transformations.

The set of numbers of cargo streams subject to the p^{th} location transformation has been defined as $\mathbf{R}_{M}^{p} = \{r \in \mathbf{R} : rm(p, r) = 1\}, p \in \mathbf{PM}.$

 $^{^{4}}$ In practice, the standard storage time parameter is used only for storage processes, for other time transformations its value may be taken as equal to 0.

Relations between cargo stream transformations with regard to form (PP), time (PT) and location (PM) are:

$$P = PP \cup PT \cup PM$$

$$PP \cap PT = \emptyset \land PP \cap PM = \emptyset \land PT \cap PM = \emptyset$$

The volume of logistic tasks accomplished within a logistic system depends on both the quantity of the to-be-processed cargo streams of individual categories described by the QZ array and the scope of the-to-be-accomplished transformations of the streams (the P set). Hence the logistic task ZL assigned to a given logistic system has been defined as:

$$\mathbf{ZL} = \langle \mathbf{QZ}, \mathbf{P} \rangle \tag{2}$$

where:

QZ – number of cargo streams submitted for processing within the system,

P – set of cargo stream transformations accomplished within the system.

4. Logistic System Resources

Logistic process is a production process primarily involving:

- means of work (including external means of transport, internal means of transport, equipment to set up shipment units, packages),
- manpower labor (including equipment operators and blue-collar workers),
- work subject (cargo streams and related information streams).

The cargo/information stream transformations are accomplished with the use of specific means of work and a specific workforce. Taking the volume of cargo stream transformations to be accomplished within the given logistic system as a constant, the cost-effectiveness of the system depends on the deployed means of work and the hired employees, which are jointly the resources of the system.

For the needs of the research a set of the **means of work categories** used in the logistic system has been defined as:

$$S = \{s : s = 1, ..., \bar{S}\}$$

Likewise, a set of the available manpower labor categories has been defined as:

$$L = \{l : l = 1, ..., \bar{L}\}$$

Both the means of work categories and the manpower labor categories may be characterized by some technical and economical features (characteristics). Therefore, the decision problem related to selection of an optimum potential of a logistic system may be solved by indicating the categories that will produce the best costeffectiveness indicators. Characteristics of individual means of work categories include:

- $Ks_s \in \mathbb{R}^+$, $s \in S$ annual total fixed cost of ownership of the s^{th} category means of work (except for their depreciation costs),
- $cs_s \in \mathbb{R}^+$, $s \in S^-$ variable cost of operation of the s^{th} category means of work generated per unit time of processing cargo streams (e.g. per minute, per hour etc.),
- $Ns_s \in \mathbb{R}^+$, $s \in S$ investment outlays necessary to deploy the s^{th} category means of work,
- $Ts_s \in \mathcal{N}, s \in S$ expected lifetime of the s^{th} category means of work,
- $tp_s \in \mathcal{N}, s \in S$ time necessary to retool the s^{th} category means of work,
- $As_s \in \langle 0, 1 \rangle$, $s \in S$ indicator of technical availability of the s^{th} category means of work,
- $Hs_s \in \langle 0, 1 \rangle, s \in S$ work time utilization rate for the s^{th} category means of work involved in cargo stream transformation,
- $\mathbf{Rs}_s = \{r \in \mathbf{R} : rs(s, r) = 1\}, s \in \mathbf{S}$ set of cargo stream categories that may be processed by the s^{th} category means of work,
- $No_s^r = \{1, 2, ..., no_s^r, ..., No_s^r\}, s \in S, r \in Rs_s$ set of successive numbers of employees necessary to run the r^{th} category cargo streams with the s^{th} category mean of work,
- $Lo_{s,no_s^r}^r = \{l \in \mathbf{L} : lo(s, r, no_s^r, l) = 1\}, s \in S, no_s^r \in No_s^r, r \in Rs_s \text{set of available}$ manpower categories for the $no_s r^{th}$ employee operating the r^{th} category of cargo streams with the s^{th} category means of work and,
- $PPs_s = \{p \in PP : Rp^p \cup Rk^p \subseteq Rs_s\}, s \in S$ set of cargo stream transformations possible to be accomplished using the s^{th} category means of work,
- $Np_s^p = \{1, 2, ..., np_s^p, ..., Np_s^p\}, s \in S, p \in PPs_s$ set of successive numbers of employees necessary to run the p^{th} form transformation with the s^{th} category means of work,
- $Lp_{s,np_s^p}^p = \{l \in L : lp(s, p, np_s^p, l) = 1\}, s \in S, np_s^p \in Np_s^p, p \in PPs_s \text{set of}$ available manpower categories for the np_sp^{th} employee operating the p^{th} form transformation with the s^{th} category means of work.

The above defined individual characteristics of the available means of work may be put together as the following F_S set:

$$\boldsymbol{F}_{\boldsymbol{S}} = \left\{ Ks_{s}, cs_{s}, Ns_{s}, Ts_{s}, tp_{s}, As_{s}, Hs_{s}, \boldsymbol{R}s_{s}, \boldsymbol{PPs}_{s}, \boldsymbol{No}_{s}^{r}, \boldsymbol{Lo}_{s,no_{s}^{r}}^{r}, \boldsymbol{Np}_{s}^{p}, \boldsymbol{Lp}_{s,np_{s}^{p}}^{p}) : s \in \boldsymbol{S} \right\}$$

The range of manpower labor characteristics is similar to that of the means of work. However, the adopted set of characteristics describing the category of the means of work includes the labor categories whose representatives may operate equipment of that category. Among all workers employed in a logistic system to accomplish cargo stream transformations there are operators of the means of work, while the others are blue-collar workers. Therefore the labor characteristics include:

• $K_{L_l} \in \mathbb{R}^+$, $l \in L$ – annual total fixed cost of labor delivered by the l^{th} category worker (employment contract salaries, cost of periodic medical check-ups etc.),

- $cL_l \in \mathbb{R}^+$, $l \in L$ variable costs of labor delivered by the l^{th} category wage worker remunerated hourly or a salaried worker additionally remunerated per hour, generated per unit of time (per hour, per day etc.),
- $N_{L_l} \in \mathbb{R}^+$, $l \in L$ recruitment/initial health check-up/training costs for the l^{th} category worker,
- $T_{L_l} \in \mathcal{N}, \ l \in L$ predicted length of employment period for the l^{th} category worker,
- $AL_l \in \langle 0, 1 \rangle$, $l \in L$ work readiness indicator for the l^{th} category worker,
- $H_{L_l} \in \langle 0, 1 \rangle$, $l \in L$ work time utilization rate for the l^{th} category worker engaged in cargo stream transformation,
- $LF = \{l \in L : lf(l) = 1\}$ set of blue-collar worker labor categories,
- $\mathbf{R}L_l = \{r \in \mathbf{R} : rl(l, r) = 1\}, l \in \mathbf{L}$ set of cargo stream categories that may be processed by workers of the l^{th} labor category.

The above defined individual characteristics of the available manpower labor categories may be put together as the following F_L set:

$$\boldsymbol{F}_{\boldsymbol{L}} = \{ K_{Ll}, C_{Ll}, N_{Ll}, T_{Ll}, A_{Ll}, H_{Ll}, \boldsymbol{R}_{Ll} : l \in \boldsymbol{L} \}$$

Likewise, set of work shifts has been defined as:

$$\mathbf{Z}\mathbf{M} = \{zm : zm = 1, ..., \overline{ZM}\}$$

Characteristics of individual work shifts include:

- $tr_{zm} \in \mathbb{R}^+$, $zm \in \mathbb{Z}M$ the zm^{th} work shift start time,
- $tz_{zm} \in \mathbb{R}^+$, $zm \in \mathbb{Z}M$ the zm^{th} work shift finish time.

The above defined characteristics of the available work shifts may be put together as the following F_{ZM} set:

$$F_{ZM} = \{tr_{zm}, tz_{zm} : zm \in ZM\}$$

Taking the above into account, the types of the technical and human resources and of the organization solutions which can be used in the logistic system has been defined as:

$$ZO = \langle \mathbf{L}, F_L, S, F_S, ZM, F_{ZM} \rangle$$

5. Logistic System Structure

Taking into consideration the fact that cargo/information streams in logistic systems are processed by means of transformations of the above specified categories, it has been assumed that some functional areas in which the transformations necessary for processing cargo streams are accomplished may be distinguished within each logistic system. The areas in which form and time transformations take place are usually organized in separated areas. In practice, the location transformations take place as moves between those areas. According to that, some areas that may be treated as basic elements of the given logistic system may be separated in each system⁵. Cargo stream form and time transformations take place in those areas. A set of areas distinguished in a logistic system may be defined as:

$$\boldsymbol{W} = \left\{ w, w' : w, w' = 1, ..., \overline{W} \right\}$$

It has been assumed that logistic system areas in which cargo stream form transformations are accomplished are separate from the areas in which cargo stream time transformations take place.

The defined wp mapping transforms individual elements of the W set into the number of 1 or 2:

$$wp: W \rightarrow \{1, 2\}$$

The transformation value wp(w) = 1 if the cargo stream form transformations are accomplished within the w^{th} area of the system, whereas wp(w) = 2 if the cargo stream time transformations are accomplished within the w^{th} area of the system.

It allows WP/WT to be defined as sets of logistic system functional areas, in which cargo stream form and time transformations are accomplished, respectively:

$$WP = \{w \in W : wp(w) = 1\}$$
$$WT = \{w \in W : wp(w) = 2\}$$

Of course, the above defined sets are mutually interrelated:

$$W = WP \cup WT$$
 and $WP \cap WT = \emptyset$

Elements that enable the outside environment to influence the logistic system in question have been distinguished within the system's formal description. These elements are referred to as the cargo stream sources. A set of cargo stream sources processed within the logistic system has been defined as:

$$\mathbf{Z} = \left\{ z : z = 1, ..., \overline{Z} \right\}$$

It has been assumed that elements of W set and Z sets jointly make up the AS set of logistic system elements:

$$AS = Z \cup W = Z \cup WP \cup WT \tag{3}$$

Taking into consideration the defined categories of logistic system elements, the system structure GS may be formally written as:

$$GS = \langle AS, RS \rangle \tag{4}$$

⁵ Functional areas intended for accomplishment of cargo stream form and time transformations.

where:

AS – set of logistic system elements,

RS – set of relations between logistic system elements.

For the needs of this paper the **RS** set has been decomposed into (i) **RS**_{ZW} set of relations between the sources of cargo streams processed in the system and the areas distinguished in it, and (ii) **RS**_{WW} set of relations between the areas distinguished within the system. The sets have been defined as:

$$RS_{ZW} = \{(z, w) \in Z \times W\}$$
$$RS_{WW} = \{(w', w) \in W \times W : w \neq w'\}$$

Of course:

$$RS = RS_{ZW} \cup RS_{WW}$$

It has been assumed that for a fixed structure of relations between logistic system elements (the **RS** set) the predecessors and successors of individual system elements are known. A set of predecessors and successors in individual areas distinguished within the logistic system have been denoted as Γ_w^{-1} and Γ_w , respectively. Due to their specificity, cargo stream sources have only successors, i.e. ΓZ_z sets.

A set of relations between system elements has been decomposed into (i) a set of functional areas in which location transformations are accomplished, and (ii) a set of formal relations. It has also been assumed that all relations between cargo stream sources and areas distinguished within the system are the formal ones.

The defined *rf* mapping transforms individual elements of the RS_{WW} set of relations between areas distinguished within the system into 0 or 1:

$$rf: \mathbf{RS}_{WW} \rightarrow \{0, 1\}$$

The transformation value rf(w', w) = 0 if some cargo stream location transformations are accomplished between the w^{th} and w'^{th} area of the logistic system, whereas rf(w', w) = 1 if both areas are connected by a formal relation only, which maps the sequence of cargo stream processing in successive logistic system areas.

Hence a set of relations between the logistic system elements that map cargo stream location transformations has been formally defined as:

$$RM = \{(w', w) \in RS_{WW} : rf(w', w) = 0\}$$

whereas a set of formal relations between the logistic system elements has been defined as:

$$RF = RS_{ZW} \cup \{(w', w) \in RS_{WW} : rf(w', w) = 1\}$$

The above defined sets are related as follows:

$$RS = RM \cup RF$$
 and $RM \cap RF = \emptyset$

The adopted logistic system structure description along with decomposition of both the set of its elements and the set of relations between the elements helps to fully represent the specifics of the modeled system.

6. Characteristics of Logistic System Structure Elements

As it results from the previous chapter, the following categories of logistic system structure elements have been identified:

- cargo stream sources,
- logistic system areas, in which cargo stream form transformations are accomplished,
- logistic system areas, in which cargo stream time transformations are accomplished,
- relations between the logistic system areas that represent cargo stream location transformations,
- formal relations between the logistic system elements.

Each of the above listed categories is different, therefore it must be described by specific characteristics.

Both the operation of submitting a cargo stream to be processed in the logistic system and the stream transformation processes themselves need some time to complete. A set of time moments in which the system is analyzed has been defined as:

$$\boldsymbol{T} = \left\{ t : t = 1, ..., \overline{T} \right\}$$

The cargo stream **source characteristics** include categories of cargo streams generated by the given source and volumes of cargo streams submitted for processing at subsequent time moments. After the introduction of some transformations the characteristics have been defined as follows:

- the set $\mathbf{R}\mathbf{z}_z = \{r \in \mathbf{R} : r\mathbf{z}(z,r) = 1\}, z \in \mathbb{Z}$ of categories of cargo streams submitted by the z^{th} source for processing within the logistic system,
- the array $QZ = [q z_z^{r,t} \in \mathcal{N} : z \in Z, r \in Rz_z, t \in T]$, in which each element $qz_z^{r,t}$ is interpreted as the volume of the r^{th} category cargo submitted for processing by the z^{th} source at time t.

Accordingly, the set of cargo streams source characteristics is given by:

$$\boldsymbol{F}_{\boldsymbol{Z}} = \{\boldsymbol{R}\boldsymbol{Z}_{z}, \boldsymbol{q}\boldsymbol{Z}_{z}^{r,t}\}$$

Characteristics of logistic system areas, in which cargo stream form transformations are accomplished include categories of processed cargo streams, capacities of places where cargo streams wait to be transformed, categories of available means of work, categories of available manpower labor (including categories of labor alternative to the technical means), categories of available cargo stream form transformations, probabilities of transformation by cargo stream category, number of transformations accomplished simultaneously with the use of available equipment or by blue-collar workers of individual labor categories, as well as times when individual transformations are being accomplished with the use of available equipment or by blue-collar workers of individual labor categories. Formally, the above characteristics have been noted as follows:

- $\mathbf{R}\mathbf{w}_w = \{r \in \mathbf{R} : r\mathbf{w}(w, r) = 1\}, w \in \mathbf{W}$ set of cargo stream categories transformed in the w^{th} area of the logistic system,
- $Pw_w^r \in \mathcal{N}, w \in W, r \in Rw_w$ capacity of places where cargo streams wait to be processed in w^{th} area of the logistic system expressed as an acceptable number of cargo stream units of the r^{th} category,
- $Sw_w = \{s \in S : sw(w, s) = 1\}, w \in W$ set of means of work categories used to accomplish cargo stream transformations in the w^{th} area of the logistic system,
- $Lw_w = \{l \in L : lw(w, l) = 1\}, w \in W$ set of manpower labor categories whose members may transform cargo streams in the w^{th} area of the logistic system,
- $Fw_w = \{l \in LF : fw(w, l) = 1\}, w \in W$ set of blue-collar worker categories that may be used as an alternative to the means of work in the w^{th} area of the logistic system to accomplish cargo transformations,
- $Zw_w = \{zm \in ZM : zw(w, zm) = 1\}, w \in W$ set of work shifts for the w^{th} area of the logistic system,
- $PPw_w = \{p \in PP : pp(w, p) = 1\}, w \in WP$ set of cargo stream form transformations categories accomplished in the w^{th} area of the logistic system,
- $Pws_{w,s} = \{p \in PPw_w : Rp^p \cup Rk^p \subseteq Rs_s\}, w \in WP, s \in Sw_w \text{set of cargo stream form transformations categories for <math>w^{th}$ area of the logistic system using the s^{th} category mean of work,
- $PwL_{w,l} = \{p \in PPw_w : Rp^p \cup Rk^p \subseteq RL_l\}, w \in WP, l \in Fw_w$ set of cargo stream form transformations categories for w^{th} area of the logistic system using the l^{th} category blue-collar worker,
- $p_w^{p,r} \in \langle 0,1 \rangle$, $w \in WP$, $p \in PPw_w$, $r \in Rp^p$ probability that the r^{th} category cargo units will undergo the p^{th} form transformations in the w^{th} area of the logistic system,
- $N_{PS_{w,s}^{p}} \in \mathcal{N}, w \in WP, s \in Sw_{w}, p \in Pws_{w,s}$ number of cargo stream form transformations of the p^{th} category that may be simultaneously executed in the w^{th} area of the logistic system using the s^{th} category mean of work,
- $NPL_{w,l}^{p} \in \mathcal{N}, w \in WP, l \in Fw_{w}, p \in PwL_{w,l}$ number of cargo stream form transformations of the p^{th} category that may be simultaneously executed in the w^{th} area of the logistic system by the l^{th} category blue-collar workers,
- $t_{PS_{w,s}^{p}} \in \mathbb{R}^{+}$, $w \in WP$, $s \in Sw_{w}$, $p \in Pws_{w,s}$ time necessary to conclude cargo stream form transformation of the p^{th} category within the w^{th} area of the logistic system using the s^{th} category mean of work,
- $tPL_{w,l}^{p} \in \mathbb{R}^{+}$, $w \in WP$, $l \in Fw_{w}$, $p \in PwL_{w,l}$ time necessary to conclude cargo stream form transformation of the p^{th} category in the w^{th} area of the logistic system using the l^{th} category blue-collar workers.

The entire set of characteristics of the logistic system functional areas in which form transformations are accomplished may be defined as:

$$F_{WP} = \{ Rw_{w}, Pw_{w}^{r}, Sw_{w}, Lw_{w}, Fw_{w}, Zw_{w}, PPw_{w}, Pws_{w,s}, PwL_{w,l}, p_{w}^{p,r}, NPS_{w,s}^{p}, NPL_{w,l}^{p}, tPS_{w,s}^{p}, tPL_{w,l}^{p} : w \in WP \}$$

Characteristics of logistic system areas in which cargo stream time transformations are accomplished include the categories of processed cargo streams, capacities of places where cargo streams wait to be transformed, categories of available means of work, categories of available manpower labor (including categories of labor alternative to the technical means), categories of available cargo stream time transformations, number of transformations accomplished simultaneously with the use of available equipment or by blue-collar workers of individual labor categories as well as times when individual transformations are accomplished with the use of available equipment or by blue-collar workers of individual labor categories. Formal notation of the first six of the above characteristics is identical to the notation given above for form transformations. The remaining characteristics have been formally noted as follows:

- $pt_w \in PT$, $w \in WT$ number of cargo stream time transformations accomplished in the w^{th} area of the logistic system,
- $NTS_{w,s}^r \in \mathcal{N}, w \in WT, s \in Sw_w, r \in Rs_s \cap Rw_w$ number of time transformations of the r^{th} category cargo streams that may be simultaneously executed in the w^{th} area of the logistic system using the s^{th} category means of work,
- $NTL_{w,l}^r \in \mathcal{N}, w \in WT, l \in Fw_w, r \in RL_l \cap Rw_w$ number of time transformations of the r^{th} category cargo streams that may be simultaneously executed in the w^{th} area of the logistic system by the l^{th} category blue-collar workers,
- $tTS_{w,s}^r \in \mathbb{R}^+$, $w \in WT$, $s \in SW_w$, $r \in RS_s \cap RW_w$ time necessary to conclude time transformation of the r^{th} category cargo stream in the w^{th} area of the logistic system using the s^{th} category means of work,
- $tTL_{w,l}^r \in \mathbb{R}^+$, $w \in WT$, $l \in Fw_w$, $r \in RL_l \cap Rw_w$ time necessary to conclude time transformation of the r^{th} category cargo stream in the w^{th} area of the logistic system using the l^{th} category blue-collar workers.

The entire set of characteristics of the logistic system functional areas in which time transformations are accomplished may be defined as:

$$F_{WT} =$$

 $\{\mathbf{R}\mathbf{W}_{w}, \mathbf{P}\mathbf{W}_{w}^{r}, \mathbf{S}\mathbf{W}_{w}, \mathbf{L}\mathbf{W}_{w}, \mathbf{F}\mathbf{W}_{w}, \mathbf{Z}\mathbf{W}_{w}, pt_{w}, NTS_{w,s}^{r}, NTL_{w,l}^{r}, tTS_{w,s}^{r}, tTL_{w,l}^{r}: w \in \mathbf{WT}\}$

Characteristics of relations that represent cargo stream moves between the logistic system areas include the categories of processed cargo streams, probabilities of transitions of the next types of loads between individual logistic system areas, capacities of places where cargo streams wait to be transformed, types of available technical means, types of the available workforce categories, types of workforce categories as an alternative to technical means, types of accomplished location transformations of the cargo stream, number of transformations accomplished with the use of available equipment and workforce and blue-collar workers as well as times when individual transformations are accomplished with the use of available equipment or by blue-collar workers of individual labor categories. Formally the above characteristics have been noted as follows:

- $R_{R_{w',w}} = \{r \in R : r_R(w', w, r) = 1\}, (w', w) \in RS \text{set of categories } r \text{ of cargo} streams which after processing in the w'th area may be directed to the wth area of the logistic system,$
- $p_{w',w}^r \in \langle 0, 1 \rangle$, $(w',w) \in RS$, $r \in RR_{w',w}$ probability that the r^{th} category cargo processed in the w^{th} area will be directed to the w^{th} area of the logistic system,
- $PM_{w',w}^r \in \mathcal{N}, (w',w) \in RM, r \in RR_{w',w}$ capacity of places where cargo streams wait to be moved from the w'^{th} area to the w^{th} area of the logistic system expressed as an acceptable number of cargo units of the r^{th} category,
- $SM_{w',w} = \{s \in S : sM(w', w, s) = 1\}, (w', w) \in RM$ set of categories of means of work that may be used to move cargo from the w'th area to the wth area of the logistic system,
- $LM_{w',w} = \{l \in L : lM(w', w, l) = 1\}, (w', w) \in RM$ set of labor categories whose members may move cargo from the w'^{th} area to the w^{th} area of the logistic system,
- $F_{M_{w',w}} = \{l \in LF : f_M(w', w, l) = 1\}, (w', w) \in RM$ set of blue-collar worker categories that may be used alternatively to the means of work to move cargo from the $w^{,th}$ to the $w^{,th}$ area of the logistic system,
- $\mathbb{Z}_{M_{w',w}} = \{zm \in \mathbb{Z}M : zM(w', w, zm) = 1\}, (w', w) \in \mathbb{R}M \text{set of work shifts for location-transformed between w'th and wth logistic system areas,$
- $pm_{w',w} \in PM$, $(w', w) \in RM$ number of cargo stream units location-transformed between w'^{th} and w^{th} logistic system areas,
- $NMS_{w',w,s}^r \in \mathcal{N}$, $(w',w) \in RM$, $s \in SM_{w',w}$, $r \in RS_s \cap RR_{w',w}$ number of the r^{th} category cargo units that may be simultaneously moved between $w^{,th}$ and $w^{,th}$ logistic system areas with the use of the $s^{,th}$ category means of work,
- $N_{ML_{w',w,l}} \in \mathcal{N}$, $(w',w) \in RM$, $l \in F_{M_{w',w}}$, $r \in R_{L_l} \cap R_{R_{w',w}}$ number of the r^{th} category cargo units that may be simultaneously moved between w'^{th} and w^{th} logistic system areas by the l^{th} category blue-collar worker,
- $tMs^{r}_{w',w,s} \in \mathbb{R}^{+}$, $(w',w) \in \mathbb{R}M$, $s \in SM_{w',w}$, $r \in \mathbb{R}s_{s} \cap \mathbb{R}R_{w',w}$ time necessary to move the r^{th} category cargo stream between w^{th} and w^{th} logistic system areas with the use of the s^{th} category means of work,
- $t_{ML_{w',w,l}^r} \in \mathbb{R}^+$, $(w',w) \in \mathbf{RM}$, $l \in \mathbf{F}_{M_{w',w}}$, $r \in \mathbf{RL}_l \cap \mathbf{RR}_{w',w}$ time necessary to move the r^{th} category cargo stream between $w^{,th}$ and w^{th} logistic system areas by the l^{th} category blue-collar worker.

The entire set of characteristics of the logistic system functional areas in which location transformations are accomplished may be defined as

$$F_{RM} = \{ RR_{w',w}, PM_{w',w}^{r}, SM_{w',w}, LM_{w',w}, FM_{w',w}, ZM_{w',w}, pm_{w',w} \\ p_{w',w}^{r}, NMS_{w',w,s}^{r}, NML_{w',w,l}^{r}, tMS_{w',w,s}^{r}, tML_{w',w,l}^{r} : (w',w) \in RM \}$$

Characteristics of formal relations between the logistic system elements include only categories of cargo streams that may be moved between the elements and the probabilities of individual moves. Both these characteristics are identical to those given above for relations that represent moves between logistic system areas. Therefore, the entire set of characteristics of formal relations between the logistic system elements may be defined as:

$$F_{RF} = \{ RR_{w',w}, p_{w',w}^r : (w',w) \in RF \}$$

The set of all the characteristics of elements of a logistic system structure is the sum of all the above defined sets:

$$FS = F_Z \cup F_{WP} \cup F_{WT} \cup F_{RM} \cup F_{RF}$$

7. Conclusions

As presented in this paper, the author's approach to the problem of mapping the flow of cargo streams through logistic systems and to the problem of mapping the transformations the streams undergo in such systems may be used, due to its general nature, as a basis to work out detailed (so-called integrated) mathematical models of various classes. The models may be expanded to include some stochastic parameters in order to better represent the modeled logistic systems. On the other hand, some parameters and/or characteristics of the general approach may be omitted within a detailed model in order to arrive at a more efficient computational algorithm. Additionally, each detailed model depends on specific principles of accomplishing logistic tasks ZR built into the model in the form of restrictions and evaluation criteria.

The formal notation presented in this paper has been utilized by author to work out:

- simulation model of logistic processes with a fixed step [13], [25], [29],
- event-driven simulation model of logistic processes [27],
- event-driven simulation model for selection of logistic system potential,
- static model for optimization of logistic system equipment [26], [28],
- static model for optimization of logistic system potential,
- dynamic model for optimization of logistic system potential.

Simulation models take into account the stochastic nature of the process of the arrival of cargo streams at the logistic system inputs as well as of the processing itself. The first and the second of the above listed models enable the user to match logistic system equipment to the to-be-accomplished tasks. Human resources and work shifts have been directly taken into consideration within the third model (whereas in the previous models those parameters are taken into account indirectly, i.e. within other system characteristics).

Static models have been built to quickly answer the question what parameters of a logistic system are necessary to enable a system to accomplish specific tasks. In consequence, the stochastic nature and dynamics of logistic processes have not been taken into consideration in them. Due to additional omitting of differentiating the system resources into manpower labor and means of work, the form of the equipment optimization model is simplified to a maximum extent and the model itself is extremely efficient from the computational point of view. On the other hand, the logistic system potential optimization model allows more precise conclusions to be drawn, however at the cost of being slightly more complex in the computational sense.

To limit computational complexity, the static and simulation models of logistic system potential have been combined into a dynamic model. The static model is used to find an initial solution, whereas the simulation model looks for a suboptimal solution. Some mechanisms built into the latter model improve the solution in subsequent simulation iterations.

The above models, their computer applications, verification elements, and calculation examples have been presented in numerous scientific papers and publications (e.g. [13], [25], [26], [27], [28], [29]).

The model proposed in this paper differs from the logistic system models by other authors (see e.g. [16, p. 370-373], [11, p. 11.16-11.17], [12, p. 142-147], [22, p. 15.3-15.5], [24, p. 703-711, 717-728], [30, p. 394-397]) by taking into consideration a number of essential aspects of real logistic systems, e.g.:

- specifics of cargo stream time, form and location transformations (e.g. formal description of cargo stream form transformations in which two streams of various categories are transformed into two streams of different categories),
- detailed mapping of logistic system resources, which are mainly the means of work and employees of various labor categories (employees split into equipment operators and blue-collar workers). What is important, the capability to define the work stations where investments goods must be operated by operators in order for the cargo stream transformations to be accomplished has been taken into consideration,
- mapping of logistic system structure taking into consideration its functional areas in which cargo stream transformations are accomplished,
- the possibility to map more than one cargo stream transformation carried out simultaneously within the same functional area with use of the same equipment or by the same blue-collar worker of the same category (e.g. a fork lift moving simultaneously more than one load palette unit or a worker moving several packages simultaneously).

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