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MODELLING OF THE SHAPE OF RAILWAY TRANSITION CURVES FROM THE POINT OF VIEW OF PASSENGER COMFORT

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Abstract:

In the past, railway transition curves were not used. Instead of it, a simple connection of the straight track and circular arc was applied. Nowadays, such simplicity is not allowed due to the increasing vehicle operating velocities. It is mainly visible in the high-speed train lines, where long curves are used. The article aims to develop a new shape of railway transition curves for which passenger travel comfort will be as high as possible. Considerations in this paper concern the polynomials of 9th- and 11th-degrees, which were adopted to the mathematical model of the mentioned shape of curves. The study's authors applied a 2-axle rail vehicle model combined with mathematically understood optimisation methods. The advanced vehicle model can better assign the dynamical properties of railway transition curves to freight and passenger vehicles. The mentioned model was adopted to simulate rail vehicle movement in both cases of the shape of transition curves and the shape of circular arc (for comparison of the results). Passenger comfort, described by European Standard EN 12299, was used as the assessment criterion. The work showed that the method using the 2-axle railway vehicle model combined with mathematically understood optimisation works correctly, and the optimisation of the transition curve shape is possible. The current study showed that the 3rd-degree parabola (the shape of the curve traditionally used in railway engineering) is not always the optimum shape. In many cases (especially for the long curves), the optimum shape of curves is between the standard transition curves and the linear curvature of the 3rd-degree parabola. The new shapes of the railway transition curves obtained when the passenger comfort is taken into account result in new railway transition curves shapes. In the authors' opinion, the results presented in the current work are a novelty in optimisation and the properties assessment of railway transition curves.

Keywords: *railway transition curves, passenger comfort, computer simulation, optimisation*

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1. Introduction

In the past, railway transition curves were not used. Instead of it, a simple connection of the straight track and circular arc was applied (Gołębiowski and Kukulski, 2020), (Kukulski et al., 2019), (Kukulski et al., 2021). Nowadays, such simplicity is not allowed due to the increasing vehicle operating velocities. It is mainly visible in the high-speed train lines, where long curves are used.

The inspiration for writing this work is the work of Kufver (Kufver, 2000). He used the famous European standard elaborated and published by CEN (CEN, 2009). In standard (CEN, 2009), CEN described the percentage P_{CT} of both seating and standing passengers with discomfort feelings using the proper mathematical formula. In (Kufver, 2000), the values of percentage P_{CT} were in the form of the second-degree parabola in the function of the transition curve length. So, it was mathematically possible to find such a value of the curve length, for which the percentage P_{CT} had a minimum value.

The aim of this article is the optimisation and the shape assessment of the railway transition curves. The authors of this work used one advanced rail vehicle model, in which the whole dynamics of the vehicles were implemented. It means the complete description of the vehicle interactions with the railway track. It is worthy of mention that such an approach is still not very popular. Instead, a mathematical point is used very often to represent the whole rail vehicle. In the authors' opinion, such an idea today is not sufficient because the complete examination of vehicle dynamics is impossible. The advanced vehicle model can better assign the dynamical properties of railway transition curves to freight (Jacyna & Krześniak, 2017), (Szaciłło et al., 2021) and passenger vehicles. In such a method, the criteria for assessing transition curves are the passengers' comfort. Such criterion can be the, for instance, the wear in wheel-rail contact.

The leading author of the current work in the article (Zboiński, 1998) has presented that the tools characteristic for the railway vehicle dynamics in the shape assessment of the railway transitions may have numerous advantages. First of all, such tools can give new information not available using the traditional approach (all system elements represented by the mathematical point). It can be that the results of the simulations obtained using this method can differ from the similar results obtained, applied the classical approach. The work (Zboiński, 1998) stated that the advanced vehicle model's simulation methods might at least complement the classical method. This touches mainly the new high-speed railway lines. For such lines, the types of rail vehicles operating are strictly specified. Numerical studies can also find a justification because, today, there are many software packages for the automatic generation of the equations of motion for rail vehicles. The programs of the lead author of this work for the fast construction of mathematical models of these vehicles can also be considered.

Up-to-date, in railway engineering, as the transition curve, the engineers use the 3rd-degree (cubic) parabola (Esveld, 2001). The fundamental formula for this curve has the following form:

$$y(x) = \frac{x^3}{6C}, \quad (1)$$

In formula (1), each variable has the following meaning:

y – transition curve offset [m],

x – independent variable, being, in fact, the current curve length [m],

C – constant being the product of radius of circular arc R, and the total curve length l_0.

Typical for this curve is that the curve (1) has the linear curvature in a function of the current curve length because the simplification – curvature $k=d^2y/dx^2$ (Esveld, 2001) – is used. In the case of the exact formula for the curvature, the linearity of the curvature has no place at the end of the curve.

2. Literature review – state of the art

The sustainable transport plays an important role in a modern world (Urbaniak et al., 2019). Scientists focus on the solutions, which may improve the efficiency of the transport systems (Gołębiowski et al., 2020), (Jacyna-Gołda et al., 2014) and reduce the fuel consumption (Kisilowski & Zalewski, 2021), (Pielecha, 2021). The examples of such works may be: (Izdebski & Jacyna, 2021), (Makarova et al., 2021). A large number of works dealing with the search for new shapes of railway transition curves can be observed also today. It is visible, especially in the context of new high-speed train lines construction (Xu et al., 2019).

Generally, three groups of the works can be distinct if we study literature, which touches the problem of

railway transition curves. A certain number of the works can also be shown as the typical works for each group.
Tari and Baykal (Tari & Baykal, 2005) represent the first group. In mentioned work, they described the idea of the lateral change of acceleration (LCA) of rail vehicles while negotiating the transition as the crucial criterion for evaluating journey comfort. The continuity in time of the function of lateral change of acceleration function must be absolutely satisfied according to them. The authors of (Tari & Baykal, 2005) searched only for the transitions, which can satisfy the mentioned imposed condition.
The work (Long et al., 2010) is the example of the second group. The authors of (Long et al., 2010) used the advanced model of a railway vehicle for the assessment of the dynamical properties of six popular transition curves, which can be applied in railway engineering practice. The most important dynamical characteristics like:

- lateral and vertical acceleration of vehicle body,
- wheel/rail lateral and vertical forces,
- derailment coefficient

were used to compare the shapes of the transitions (Zboiński & Woźnica, 2012), (Zboiński & Woźnica, 2018). Relatively the same idea can also be found in, e.g. (Fischer, 2009), (Li et al., 2010), (Pirti et al., 2016).
The last group is the group in which the properties of transition curves are examined, assuming that the curve is the mathematical object together with all its mathematical (geometrical) properties. In this group of works, the authors focus on finding the transitions better than the 3rd degree parabola (Koc, 2019), (Shen et al., 2013). The examples of such works can also let be: (Ahmad & Ali, 2008), (Barna & Kisgyörgy, 2015), (Eliou & Kaliabetsos, 2014).
In the introduction of the current work, the authors also mentioned about work (Kufver, 2000), but this work does not constitute a separate group of the works.
Analysing the literature relating to the problem of railway transition curves, the authors of the current work may conclude that the works, where the full dynamics of the vehicle-track system and mathematically understood optimisation methods applied together, are still not very popular. The traditional (classical) approach to this problem is used up-to-date (Esveld, 2001), (Kobryń, 2014). The relatively simple rail vehicle model and the maximum value of unbalanced lateral acceleration (cant deficiency (EU, 2014)) and its change (abrupt change of cant deficiency (EU, 2014)), which should not be exceeded (as the fundamental demand) are still popular even now. The non-traditional approach in which in the transition curve shape optimisation the advanced rail vehicle model of the vehicle-track system and mathematically understood optimisation methods are applied is still a certain novelty.

3. The object, model, and the corresponding model

In the current work, the polynomials of 9th and 11th degrees were applied as the transition curve. These (high) degrees of polynomials were taken due to the bigger flexibility of curve shape modelling. General formulas for curve, curve curvature, cant, and also inclination of superelevation ramp applied is:

$$y=\frac{1}{R}\begin{bmatrix}\frac{A_n l^n}{l_0^{n-2}}+\frac{A_{n-1}l^{n-1}}{l_0^{n-3}}+\frac{A_{n-2}l^{n-2}}{l_0^{n-4}}+\frac{A_{n-3}l^{n-3}}{l_0^{n-5}}+\ldots\ldots\\ +\frac{A_4 l^4}{l_0^2}+\frac{A_3 l^3}{l_0}\end{bmatrix} \tag{2}$$

$$k=\frac{d^2y}{dl^2}=\frac{1}{R}\begin{bmatrix}n(n-1)\frac{A_n l^{n-2}}{l_0^{n-2}}+(n-1)(n-2)\frac{A_{n-1}l^{n-3}}{l_0^{n-3}}+\ldots\ldots\\ +3\cdot 2\frac{A_3 l}{l_0}\end{bmatrix} \tag{3}$$

$$h=H\begin{bmatrix}n(n-1)\frac{A_n l^{n-2}}{l_0^{n-2}}+(n-1)(n-2)\frac{A_{n-1}l^{n-3}}{l_0^{n-3}}+\ldots\ldots\\ +3\cdot 2\frac{A_3 l}{l_0}\end{bmatrix} \tag{4}$$

$$i=\frac{dh}{dl}=H\begin{bmatrix}n(n-1)(n-2)\frac{A_n l^{n-3}}{l_0^{n-2}}+\\ (n-1)(n-2)(n-3)\frac{A_{n-1}l^{n-4}}{l_0^{n-3}}+\ldots\ldots+3\cdot 2\cdot 1\frac{A_3}{l_0}\end{bmatrix} \tag{5}$$

where the meaning of each symbol is:
y – transition curve offset [m],
n – degree of the polynomial,

R – curve radius [m],
l_0 – total curve length [m],
l – curve current length [m],
k – curvature [1/m],
h – superelevation ramp (cant) [m],
i– inclination of the superelevation ramp [-].

For each function – y, k, h, and i – the fundamental geometrical demands in the extreme points of the curve – the first one and the last one – were also applied. For example, for curvature function, the mentioned (fundamental) demand is as follows:
- the value of curvature function in the first point of the curve - 0,
- the value of curvature function in the last point of the curve – $1/R$.

In the current work, only one criterion for transition curve assessment was applied by the authors. This criterion is passenger comfort, described by European Standard EN 12299 (CEN, 2009). Using the formula given in (CEN, 2009), we have the following relation:

$$P_{CT} = 100 \begin{Bmatrix} \max\left[\left(A|\ddot{y}_{\max}| + B|\dddot{y}_{\max}| - C\right);0\right] \\ +\left(D|\dot{\phi}_{\max}|\right)^{E} \end{Bmatrix}, \quad (6)$$

where:
P_{CT} – the percentage of dissatisfied passengers (comfort index on transitions),
$\ddot{y}_{\max}$ – maximum lateral acceleration of vehicle body in transition curve [m/s^2],
$\dddot{y}_{\max}$ – maximum lateral jerk of vehicle body in transition curve [m/s^3],
$\dot{\phi}_{\max}$ – maximum roll velocity of the vehicle body in transition curve [rad/s].

Formula (6) can be used both for standing and seating passengers. In Table 1, the authors of the work presented the values of the constants used in the formula (6) for both cases.

Table 1. Values of constants

Constant	Unit	**Standing**	Seating
A	s^2/m	0.2854	0.0897
B	s^3/m	0.2069	0.0968
C	-	0.111	0.059
D	s/rad	3.64	0.916
E	-	2.283	1.626

In the current work, the optimisation relied on the minimisation of the value of the comfort index from formula (6).
The lengths of the curves were calculated using the traditional method, known from the engineering practice (Esveld, 2001). The maximum values of vehicle lateral jerk (abrupt change of cant deficiency) and wheel vertical rise velocity on superelevation ramp (abrupt change of cant) were taken into account here.
Two standard polynomial curves of 9th and 11th degrees were applied as the initial curves in the optimisation process. These curves are as follows:

$$y_9 = \frac{1}{R}\left(-\frac{5}{18}\frac{l^9}{l_o^7} + \frac{5}{4}\frac{l^8}{l_o^6} - 2\frac{l^7}{l_o^5} + \frac{7}{6}\frac{l^6}{l_o^4}\right), \quad (7)$$

$$y_{11} = \frac{1}{R}\left(\frac{7}{11}\frac{l^{11}}{l_0^9} - \frac{7}{2}\frac{l^{10}}{l_0^8} + \frac{15}{2}\frac{l^9}{l_0^7} - \frac{15}{2}\frac{l^8}{l_0^6} + 3\frac{l^7}{l_0^5}\right). \quad (8)$$

The only odd polynomials were considered due to the fact that they have only one standard curve.
In the current work, one model of railway vehicle was also used. This model has a 2-axle structure. It was called a "2-axle freight (cargo) wagon" in many earlier works (e.g., in (Zboiński & Woźnica, 2012)). The model posses a body connected with two wheelsets with spring-damping elements. The structure of the model and its parameters correspond to a typical British real wagon. This model is off-course of crucial importance in this work, due to the fact that it was used to examine the dynamical properties and optimise the shape of the transitions. The nominal model of this 2-axle rail vehicle is presented in Figure 1c. The vehicle model is off-course supplemented with the model of the track. It is also presented in Figure 1, as the part (a) and (b). The whole track-vehicle system is discussed in detail in many earlier works of the authors lastly, e.g., in (Zboiński & Woźnica, 2012). The model parameters of this system are also shown in (Zboiński & Woźnica, 2012).
The approach to the transition curve shape modelling applied in the current work was widely described e.g., in (Zboiński & Woźnica, 2012). The dynamics of relative motion is applied in the mentioned approach. The definition of railway vehicle dynamics is relative to the track-based moving reference frame.

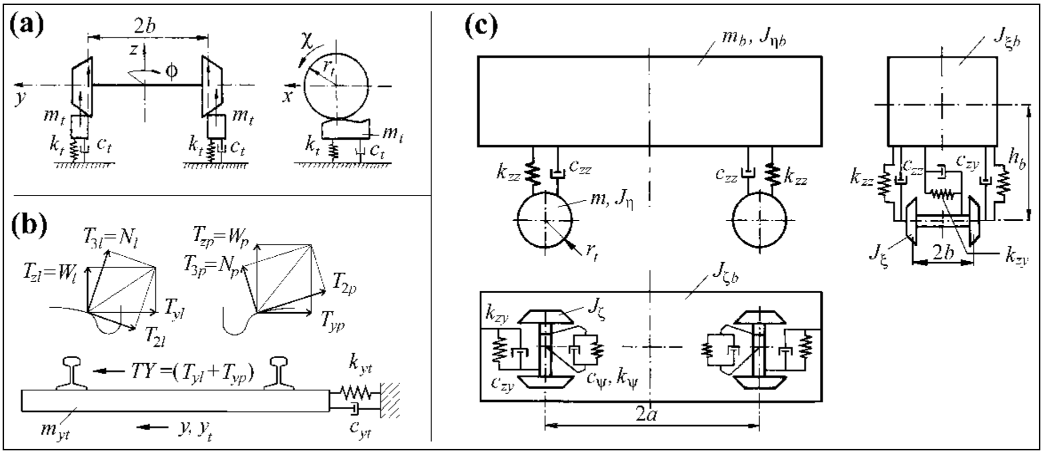

Fig. 1. 2-axle vehicle model: a) track vertically, b) track laterally, c) vehicle

The idea used in the work relied on the passage of the railway model through the constant route consisted of straight track, transition curve, and circular arc.

The software, as the scheme, is shown in Figure 2. In this Figure, two iteration loops can be visible. The first loop is the integration loop. It stopped when the assumed length of the route was reached by the rail vehicle during the simulations. The second loop was the optimisation loop. This loop stopped when the assumed value of iterations was reached, unless the optimum solution was not found earlier. The typical time of calculations was not larger than 1 hour for each optimisation process, using Intel Core Duo 2 GB processor.

4. Results of the studies

In this section, the authors of the work presented the results of the optimisation of railway transition curves when the criterion of assessment was passenger comfort. The routes of the vehicle, as mentioned, were always composed of straight track, transition curve, and circular arc.

In the optimisation performed, the length of the straight track was every time equal to 50 m, whereas the circular arc length was equal to 100 m.

The optimisation results, in general, consisted of optimum transition curves and their curvatures described by the polynomial coefficients, dynamics of the rail vehicle, and creepages in wheel-rail contact.

The parameters assumed in optimisation for two different degrees of the polynomial – 9th one and 11th one – are presented in Table 2. Mentioned parameters are:

– the radius of circular arc R,
– the cant H,
– the length l_0 of the curve,
– vehicle velocity v.

Admissible unbalanced acceleration a_{lim} on the track level was calculated for particular R, H, and velocity v, and ranged from 0.0 m/s^2 to 0.6 m/s^2.

The curvatures of the curves obtained in the work generally had five possible shapes. These shapes are as follows:

1) the standard curve shape – type 1,
2) the shape with the inflection point in the middle part of the curve – type 2,
3) the linear shape – type 3,
4) the convex shape on the route of the transition – type 4,
5) the concave shape on the route of the transition – type 5.

Figures 3 and 4 show 4 typical types of the curvatures – no. 1, 2, 4, and 5.

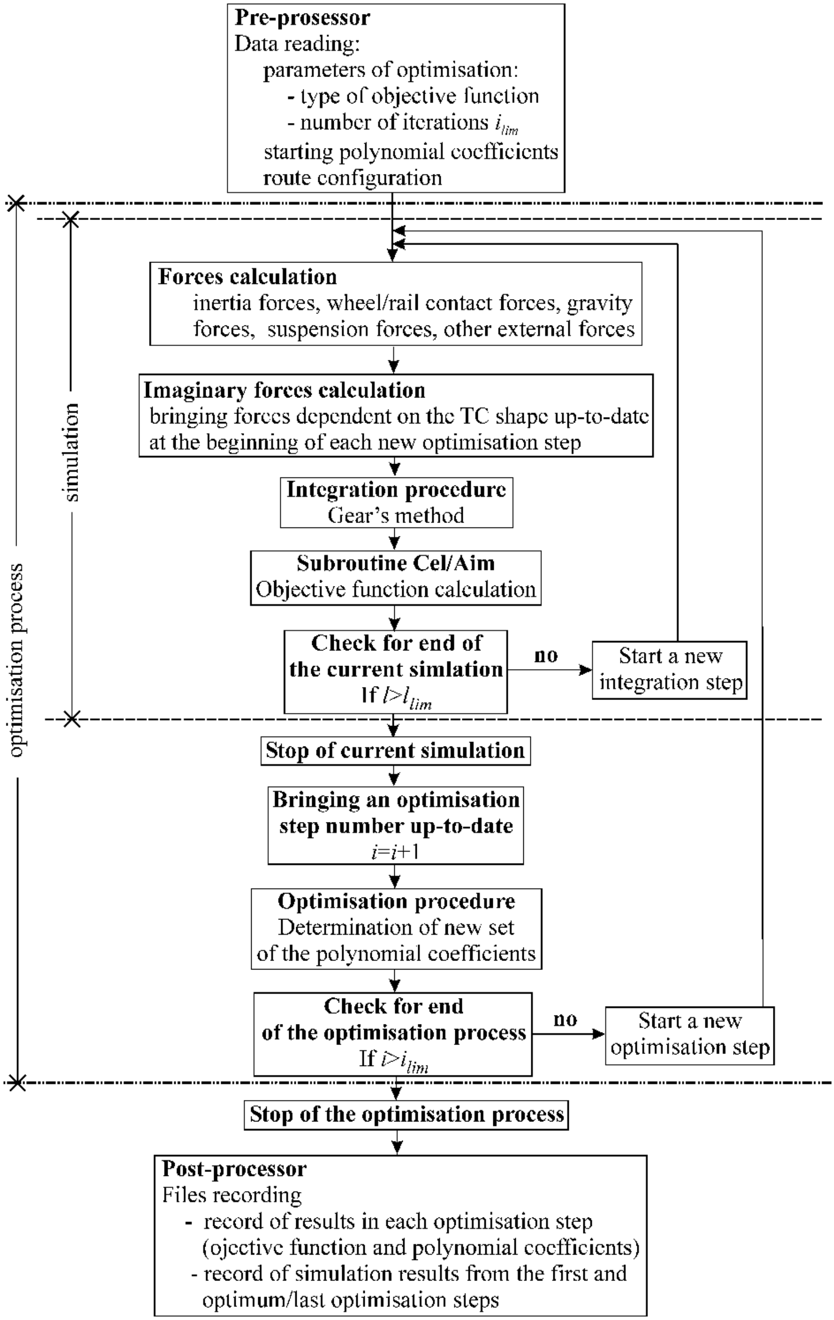

Fig. 2. The scheme of software

Table 2. Assumed parameters for 9th and 11th degrees.

Order number	**Degree**	**Radius *R*, cant *H***	**Length l_0, velocity *v***
1	9th	600 m, 150 mm	142.15 m, 24.26 m/s
2	11th	600 m, 150 mm	159.86 m, 24.26 m/s
3	9th	600 m, 150 mm	180.46 m, 30.79 m/s
4	11th	600 m, 150 mm	202.94 m, 30.79 m/s
5	9th	1200 m, 75 mm	71.07 m, 24.26 m/s
6	11th	1200 m, 75 mm	79.93 m, 24.26 m/s
7	9th	1200 m, 70 mm	82.41 m, 30.14 m/s
8	11th	1200 m, 70 mm	92.68 m, 30.14 m/s
9	9th	1200 m, 75 mm	105.98 m, 36.17 m/s
10	11th	1200 m, 75 mm	119.18 m, 36.17 m/s
11	9th	2000 m, 45 mm	42.64 m, 24.26 m/s
12	11th	2000 m, 45 mm	47.95 m, 24.26 m/s
13	9th	2000 m, 50 mm	60.29 m, 30.87 m/s
14	11th	2000 m, 50 mm	67.80 m, 30.87 m/s
15	9th	2000 m, 45 mm	60.60 m, 34.47 m/s
16	11th	2000 m, 45 mm	68.15 m, 34.47 m/s
17	9th	3000 m, 50 mm	61.13 m, 31.30 m/s
18	11th	3000 m, 50 mm	68.74 m, 31.30 m/s
19	9th	3000 m, 25 mm	29.94 m, 30.66 m/s
20	11th	3000 m, 25 mm	33.67 m, 30.66 m/s

In Table 3 the authors presented the results of the optimisations – the percentage of dissatisfied passengers and types of the curvatures – for all 20 cases for:
– for degree of polynomial 9^{th},
– for degree of polynomial 11^{th},
– for standing passengers,
– for seating passengers.

One case from this table is taken as the representative case for presenting the results of the optimisations. This case is no. 17 for single curve radius *R,* and superelevation *H* for circular arc assumed. Their values are *R*=3000 m and *H*=0.05 m, respectively.

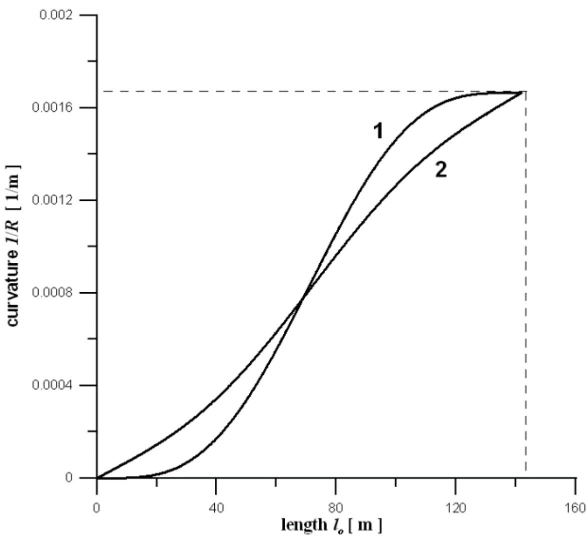

Fig. 3. The four types of curvatures: 1 and 2

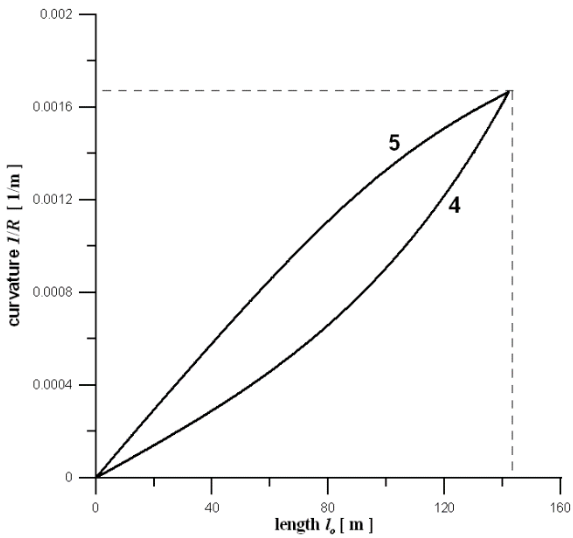

Fig. 4. The four types of curvatures: 4 and 5

Table 3. The results of the optimisation for 9th and 11th degrees

Order number	Degree	Standing	Seating
1	9th	0.008 % - 3	0.051 % - 3
2	11th	0.007 % - 4	0.088 % - 4
3	9th	0.145 % - 2	0.356 % - 2
4	11th	0.090 % - 2	0.384 % - 2
5	9th	0.010 % - 3	0.062 % - 4
6	11th	0.014 % - 3	0.067 % - 3
7	9th	0.257 % - 5	0.589 % - 5
8	11th	0.222 % - 4	0.518 % - 4
9	9th	1.151 % - 3	1.610 % - 3
10	11th	0.880 % - 3	1.353 % - 3
11	9th	7.743 % - 4	1.561 % - 4
12	11th	9.369 % - 4	4.960 % - 4
13	9th	0.282 % - 3	4.170 % - 4
14	11th	0.081 % - 3	7.070 % - 4
15	9th	9.028 % - 4	3.139 % - 4
16	11th	23.088 % - 4	3.346 % - 4
17	9th	0.149 % - 3	0.335 % - 3
18	11th	1.626 % - 4	0.329 % - 3
19	9th	7.085 % - 3	50.737 % - 4
20	11th	82.728 % - 4	20.049 % - 4

As we can see, the best situation – the lowest percentage of dissatisfied passengers – is for the longest transitions. For the shortest curves, the situation is opposite. So, the rule – the smaller length of the curve, the larger value of PCI – obeys.

The number of types of the curves are as follows:

– type no. 1 – 0,
– type no. 2 – 4,
– type no. 3 – 15,
– type no. 4 – 19,
– type no. 5 – 2,

so the types no. 3 and 4 dominated over the rest types.

Figures 5 and 6 present the types of curvatures versus the curve length.

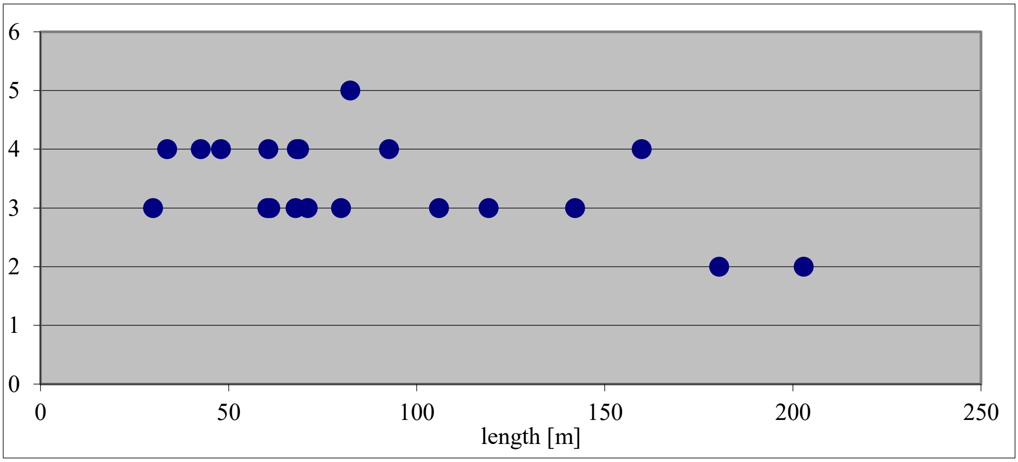

Fig. 5. Types of the curvatures versus curve length (standing)

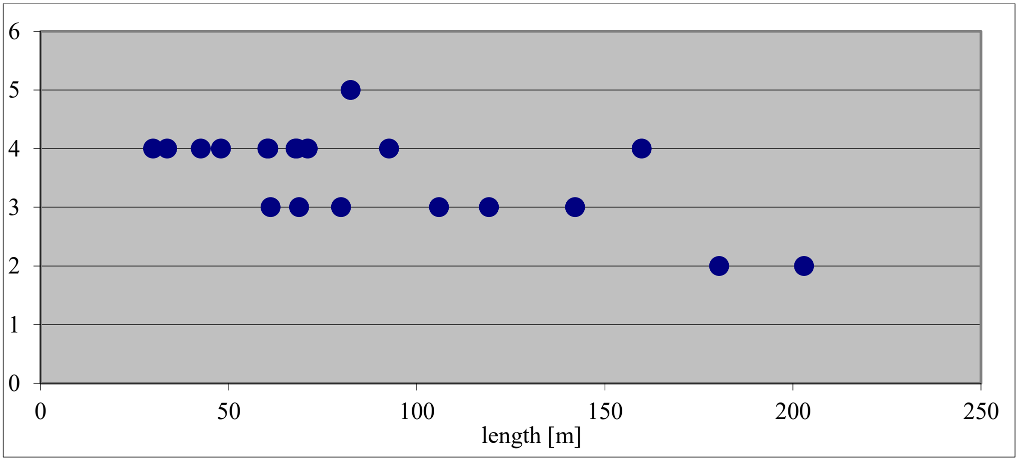

Fig. 6. Types of curvatures versus curve length (seating)

If we analyse both Figures (5 and 6), we may conclude that a certain trend of the curve types exists. This trend is 4->3->2. For the shortest curves, the type no. 4 dominates. For the longest curves, only the type no. 2 can have the chance to be the optimum transition curve.
Table 4 presents the values of optimum coefficients of the polynomial (formula (1)) with accuracy to 5 decimal places for the case no. 17 from Table 2.
The value of optimum objective function value is 0.149%, so it means only such percentage of passenger was dissatisfied. For this case, we can also calculate the ratio of the values of the integrals of vehicle body lateral accelerations along the route. This ratio is 0.006, and is significantly better for the cubic parabola.

Table 4. Optimum polynomial coefficients (case no. 17)

Order number	**Coefficient A_i**	**Value of A_i**
1	A_9	-0.27589
2	A_8	1.24102
3	A_7	-1.97283
4	A_6	1.13847
5	A_5	-0.00966
6	A_4	-0.04533
7	A_3	6.97992

The graphical results of the optimisations are also limited to the case no. 17 from Table 2. Figure 7 shows the transition curves comparison (curve offsets y) - initial one (black line), optimum one (red line). Figure 8 presents their curvatures. It can be seen that the curvature of the optimum transition curve is in fact, the linear curvature of the 3rd degree parabola.

Figure 9 presents the comparison of the lateral displacement and acceleration of the centre of mass of the car body for the initial curve and the optimum curve (the 3rd degree parabola). Both the displacement graph (Figure 9) and the acceleration graph (Figure 10) clearly show that the optimum curve is better than the standard curve. Notably, in this context, interesting is Figure 10 for accelerations.

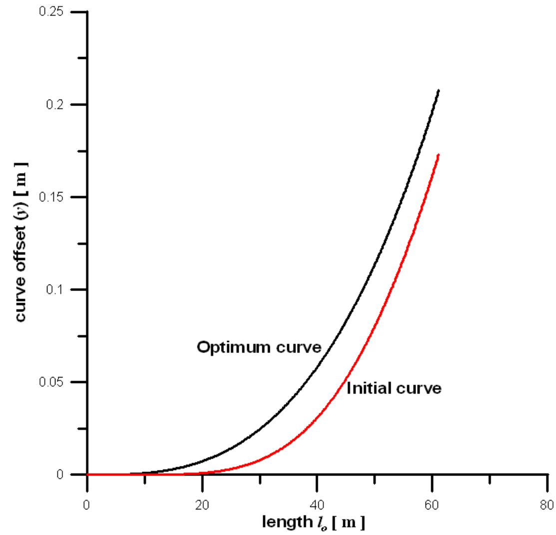

Fig. 7. Transitions curves offsets curvatures

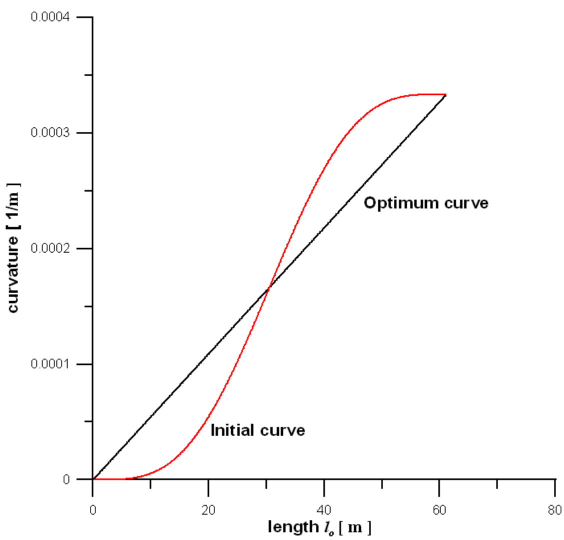

Fig. 8. Transitions curves: (a) curve offsets curvatures

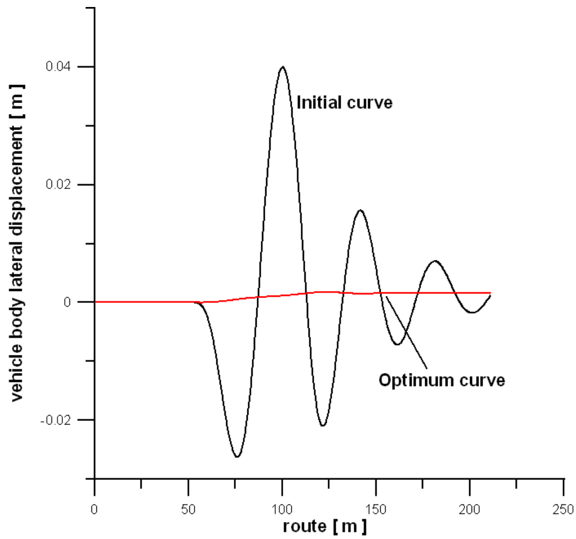

Fig. 9. Dynamical characteristics: (a) vehicle body lateral displacements

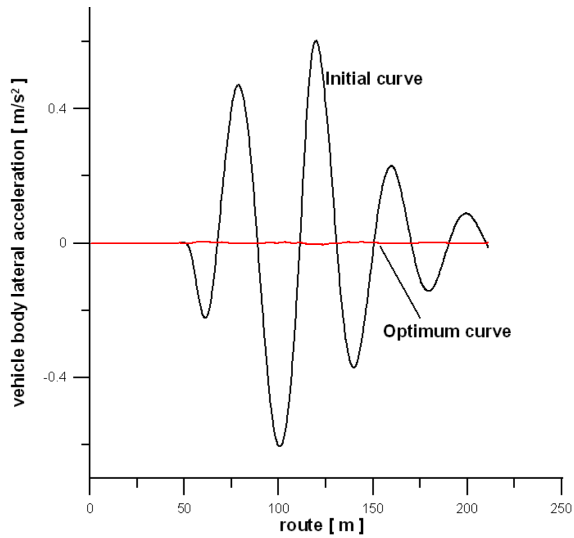

Fig. 10. Dynamical characteristics: vehicle body lateral accelerations

5. Conclusions

The work showed that the method using the 2-axle railway vehicle model combined with mathematically understood optimisation works correctly, and the optimisation of the transition curve shape is possible. The new shapes of the railway transition curves obtained when the passenger comfort is taken into account result in new railway transition curves shapes. The results presented in the current work are, in the authors' opinion a certain novelty in the field of optimisation and the properties assessment of railway transition curves.

The new curves of 9th and 11th degrees had significantly better properties than known from the literature standard curves of those degrees when, as mentioned, passenger comfort was taken as the criterion.

The work showed a certain trend of transformation of transition curve curvature in accordance with the rule 4 -> 3 -> 2. The work also showed that in the majority of cases, both for standing and seating passengers, the types of curvatures are the same. Also, in the majority of cases, the values of the Comfort Index are very small, and only for the curves with great lengths, the values had significantly large values.
The results presented in the work allow believing that this work may be a good starting point for adopting passenger comfort to assess the transition curves' shape. In this context, the authors mean to optimise the shape of the curves for the larger values of the circular arc, which are used in high-speed trains infrastructure.

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