

# AN IMPROVED MATHEMATICAL MODEL FOR VEHICLE CRASH AGAINST HIGHWAY GUARDRAILS

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**Abstract:**

Highway guardrail is a kind of important road traffic safety facility. When a vehicle is travelling on a highway, it can lose control because of accident. The guardrail can prevent the vehicle from rushing directly out of the road, so as to reduce the injury to the driver in the vehicle. Therefore, the guiding performance, anti-collision performance and buffer performance of the guardrail are important indexes to reflect the highway guardrail safety in the traffic accidents between vehicle and guardrail. The process of collisions between vehicles and guardrails is a complex motion, affected by multiple factors such as the movement patterns and types of vehicles, the types of guardrail, the bending stiffness of the beams, the speed of collision, the angle of collision, etc. The accuracy of energy estimation when vehicle collides with guardrail is the foundation of highway guardrail design, installation and improvement. Many experts and scholars at home and abroad have done a lot of theoretical research and experimental verifications on the safety performance of highway guardrail, and analyzed the anti-collision ability and energy absorption effect of highway guardrail. Single degree of freedom model is the most widely used mathematical model of vehicle collision in highway guardrail. The traditional model is more suitable for calculating the maximum impact force of small vehicles, but it is not accurate for large vehicles. However, due to the unreasonableness of the model in the theoretical derivation process, there is a large error in the mathematical model, especially in estimating the accuracy of the energy value of the large vehicle collision guardrail. Practice shows that the current guardrail cannot withstand the impact of large vehicles. Once large vehicles collide with the corrugated beam guardrail, the guardrail will collapse in most cases, and the vehicle will rush out of the road directly, so it is very difficult to exert the protective function of the guardrail. The anti-collision performance of guardrail is poor, which is related to the existing calculation model, which results in insufficient strength in the design of guardrail.

**Key words:**

roadway engineering, mechanical model, roadway guardrail, lateral rotation, vehicle crash

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## 1. Introduction

Highway guardrails are designed and installed to prevent vehicles from driving out of the road or into the opposite lane, re-direct the vehicle to right driving lanes, and reduce the risk of injury for the vehicle occupants. Based on different impacts of deformation after collision, highway guardrails have three types: rigid guardrails, semi-rigid guardrails, and flexible guardrails. Respective examples are concrete guardrails, wave beam guardrails, and cable guardrails. Wave beam guardrails have collision prevention functions with the energy-absorbing capability through deformation. Due to its low maintenance costs and aesthetic appeal, they are widely used in many countries. Concrete guardrails are widely used on roadway bridges. Cable guardrails are rarely used due to the relatively large deflections as well as high installation and maintenance costs. In the United States, design of guardrails must follow the guidelines in *The Roadside Design Guide*, which is developed by the American Association of State Highway and Transportation Officials (AASHTO, 2011). This design guide presented a synthesis of operating practices related to the roadway safety. Ross et al. (1993) developed the standard guidelines for testing the crash performance of the steel strong-post w-beam guardrails, the content of which are described in the National Cooperative Highway Research Program (NCHRP) Report 350. This standard was superseded by AASHTO's (2009) *Manual for Assessing Safety Hardware* (MASH). Guardrails are implemented to prevent vehicles from veering off roadways. However, the guardrail itself can be a severe hazard to the drivers and vehicle's occupants. It is reported that in the United States, approximately 1200 fatalities were caused by guardrails annually and that 13% of guardrail accidents caused a vehicle rollover (Wolford and Sicking, 1996). A separate study shows that over half of all the fatal collisions with guardrails involved secondary events which can be either a second impact or a rollover (Gabler and Gabauer, 2006). According to the traffic accident statistics from China, more than 30% of all accidents on freeways are related to collisions between vehicles and median barriers (China's Ministry of Public Security Traffic Management Bureau, 2013). In order to evaluate the performance of different types of guardrails, we investigated the traffic crash data collected from freeways

in Zhejiang Province, China (e.g., Hangrui Expressway, Hangchang Expressway and Hangliang Expressway). It is found that vehicle collisions with concrete guardrails in one-vehicle accidents did not cause any fatality for years. In the same period of comparison, vehicle collision with other types of guardrails in one-vehicle accidents resulted in a fatality rate of 270 people per thousand kilometers. The data shows that vehicle collisions with wave beam guardrails are more likely to cause a secondary accident, which increases the probability of occupant casualty. There are several reasons for such a phenomenon, one of that is about the guardrails standards which were developed and adopted by the developed countries such as the United States and Japan. These guardrail standards may not fit in the current traffic conditions in China. In addition, the study on vehicle collision with guardrails remains limited. The existing theoretical models are not able to provide accurate prediction and that causes guardrails cannot achieve the desired performance in prevention of crash and absorption of energy. Therefore, concrete guardrail is safer than corrugated beam guardrail. At present, the passive safety technologies of the automobile have become more and more mature, so it is unnecessary to emphasize the deformation too much. There are several reasons for such a phenomenon, one of that is about the guardrails standards which were developed and adopted by the developed countries such as the United States and Japan. These guardrail standards may not fit in the current traffic conditions in China. In addition, the study on vehicle collision with guardrails remains limited. The existing theoretical models are not able to provide accurate prediction and that causes guardrails cannot achieve the desired performance in prevention of crash and absorption of energy. One possible reason for a higher fatality rate associated with wave beam guardrails on Chinese freeways is that the guardrail standards, which were developed in the United States, may not fit in the traffic conditions in China.

In the past few decades, significant efforts have been made to develop theoretical models and simulation programs to analyze vehicle collisions with guardrails. A classic collision mechanical model to predict the maximum force of impact from the collision can be presented in the following equation (AASHTO, 1989):

$$F_{\max}^0 = \frac{\pi}{4} \cdot \frac{m(v_0 \sin \theta)^2}{c \sin \theta - b(1 - \cos \theta) + z} \quad (1)$$

where  $F_{\max}^0$  is the maximum impact force from the collision,  $m$  is the mass of the vehicle,  $v_0$  is the vehicle's approaching speed,  $\theta$  is the collision angle of the vehicle,  $c$  is the distance from the vehicle's center of gravity to its front bumper, and  $z$  is the parameter of guardrail deformation. This classic mechanical model is still widely used in evaluating the impacts of vehicle crash against guardrails. Validated with the data collected from the real-world guardrail crash testing, it was found that the level of error using the classic model to calculate the impact force can be as high as 20%. While the model is relatively accurate for small and light vehicles, the errors increase when larger and heavier vehicles were used in testing.

Hendricks and Wekezer (1996) are among the first researchers who developed a finite-element model of the weak-post wave beam guardrails and analyzed them under vehicle impact conditions. They simulated longitudinal impacts with the guardrails and produced a proper redirection and good comparisons with the maximum rail deflection and exit speed. Mackerle (2003) provided a bibliography of research between 1998 and 2002 on finite-element models for vehicle crash simulation and impact analysis. Most of the finite element models were developed using LS-DYNA, a general-purpose multiphysics simulation software package (Raghu, 2010). Ferdous et al. (2011) analyzed the performance of four types of guardrails using LS-DYNA and identified the override and under-ride limits for each guardrail model. Ray et al. (2003) performed an in-service performance evaluation for cable guardrails, weak-post W-beam guardrails and strong-post W-beam guardrails. They proposed a procedure manual for planning and implementation of in-service evaluations for roadside hardware based on the findings of the study. Hampton et al. (2010) examined the crash performance of strong-post W-beam guardrail with rail-and-post deflection from a previous impact. They found that the combination of rail-and-post deflection can negatively affect the crash performance based on the crash tests and finite element simulations of second impacts into damaged guardrail. More recently, Marzogui et al. (2015) used a finite-element simulation model to

assess vehicle trajectories when vehicles leave the traveled way on curved and super-elevated roadway sections. Their research concluded that barriers with increased heights and deflection zone should be used at roadway sections of sharper curves and super-elevation after analysis of barrier's dynamic response to the vehicle impacts.

Many experts and scholars in China have done a lot of researches and experiments on highway guardrail, and been striving to establish a more accurate mathematical model. Liu and Tang (2012) proposed mechanical models for vehicle guardrail overriding accidents and also for underriding accidents according to the law of energy conservation, and put forward high demands for wave beam guardrails or boards. Zhang et al. (2012) simplified the collisions of cars with the composite anti-collision guardrails into a combined model of rigid guardrail and spring guardrail, and established a collision mechanical model. Xiao et al. (2012) made appropriate assumptions about the collision process, and use the law of energy conservation and momentum theorem to establish a simplified calculation model of collision between vehicle and cable guardrail. Zhou et al. (2008) simulated the automobile and guardrail system with finite element modeling, using LY-DNSA calculation to calculate how the wave beam guardrail depends on the plastic deformation of the columns and the tailgates to achieve the energy absorption effect. Zhao and Liu (2012) used the dynamic display nonlinear finite element analysis software LY-DNSA to simulate respectively the rigid guardrail and the semi-rigid guardrail, and drew the conclusion that when the vehicle collides with the rigid guardrail, the drivers and riders are in a more dangerous situation. Huang et al. (2002) built a model for the car, the drivers and riders and the guardrail by using multi rigid guardrail simulation software MAD-YMO. The collision between the car and the concrete guardrail is more severe than the collision between the car and the wave beam guardrail, and more harmful to the drivers and riders. Lei Zhenbao, using VPG processing software, analyzed the vehicle collision safety impact of flexible guardrail, also the finite element simulation analysis, and found out that the flexible guardrail has a better vehicle collision guiding performance, the injury to the drivers and riders is small, it can effectively avoid collisions occurred in the process of "stumbling-block" effect

easily. Li, et al. (2014) simplified each of the vehicles and guardrail as a whole, and put the two simplified parts into a spring model, and proposed a dual model.

Although the classic mechanical model and various simulation based tools can provide practical guidelines for guardrail design, the previous models have substantial errors in calculating the impact force due to simplified assumptions of vehicle characteristics and its movement during the crash. It remains a challenging work to improve the models' accuracy in predicting the impacts of crash. In this research we developed an improved mathematical model for the purpose. The improved model can achieve much better performance for collisions with no lateral rotations. The remaining content is organized as follows: a theoretical model is formulated in the next section, followed by a comparison with the classic mechanical model which was presented in the AASHTO's Guide Specifications for Bridge Railing. The conclusions are drawn in the final section.

**2. Dynamics analysis of vehicle collision with guardrail**

The process of the vehicle and guardrail collision can be divided into two stages,  $t_1$  and  $t_2$ , where stage  $t_1$  begins at the time when the vehicle loses its control and collide with the guardrail and ends at time just before the rotation starts. Stage  $t_2$  starts at the time when the vehicle begins to rotate caused by the collision and ends when vehicle's rotation stops.  $t_1$  is the compression stage of the collision, while  $t_2$  is the recovery stage of the collision.

**2.1. Analysis of the force in stage  $t_1$**

The process of vehicle collision with a guardrail is illustrated in Figure 1.

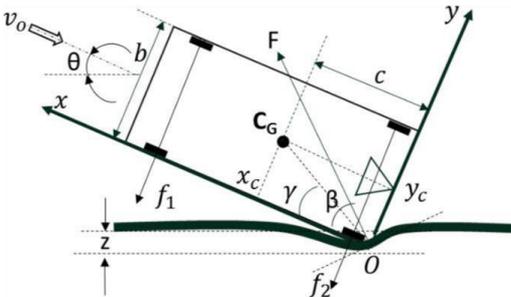


Fig. 1. Force analysis of vehicle collision with guardrail

In the  $t_1$  stage, the guardrail produces a reaction force on the contact,  $F$ , as shown in Figure 1 after deformation occurs in the vehicle crash. A rectangular coordinate system can be established as shown in figure. Ignoring the friction between vehicle's tires and the ground and the tripping resistance of the vehicle body and the guardrail, the kinetic equations of the vehicle satisfy:

$$I_x = \int_0^{t_1} F_x \cdot dt \tag{2}$$

$$I_y = \int_0^{t_1} F_y \cdot dt \tag{3}$$

$$L = \int_{t_1}^{t_2} (M_x - M_y) \cdot dt \tag{4}$$

where :

$F_x, F_y$  - Components of force  $F$  in  $x, y$  directions (N);

$M_x, M_y$  - Torque of the vehicle in (kg. m);

$I_x, I_y$ : - Impulses of the vehicle; L: Angular impulse of the vehicle.

**2.2. Analysis of motion at stage  $t_2$**

In stage  $t_2$ , the vehicle's speed decreases along the negative direction of the  $x$  under the action of elastic force  $F$ , and there is a tendency of rotation and lateral movement. For small vehicles with light weight, lateral rotation likely occurs after the collision. However, lateral rotation does not occur for heavy and large vehicles. The criteria to determine whether lateral rotation will occur can be expressed as:

- 1) If  $M_x = M_y$ , rotation will not occur. In this scenario, angle  $\gamma = \beta$ , and the elastic force  $F$  passes through the center of gravity of the car.
- 2) If  $M_x < M_y$ , it means  $\gamma < \beta$ , the vehicle has a tendency to rotate counterclockwise.
- 3) If  $M_x > M_y$ , it means  $\gamma > \beta$ , the vehicle has a tendency to rotate clockwise.

We have  $\tan \gamma \approx \frac{b}{2c}$ , where  $b$  is the width of the

car,  $c$  is the distance between the center of gravity and the front bumper. Angle  $\gamma$  is small for the large and heavy vehicles and is big for small vehicles. The value of  $\beta$  is jointly determined by the degree of guardrail deformation and the contact surface of the vehicle. For a concrete barrier,  $\beta \rightarrow 90^\circ$ . The motion of vehicle in state  $t_2$  depends on the vehicle types and

deformation of guardrails. The tendency of rotation for different vehicles and different guardrails is summarized in Table 1.

Table 1. Rotation tendency for different vehicle and guardrail types

Guardrail Type	Large and Heavy Vehicle	Small and Light Vehicle
Rigid (concrete)	The rear part of vehicle rotates anticlockwise in small angle, the vehicle body moves in parallel with the un-deformed guardrail, no lateral rotation occurs.	The rear part of vehicle rotates anticlockwise in small angle, the vehicle body move in parallel with the un-deformed guardrail, no lateral rotation occurs.
Semi-rigid (W-wave beam)	The rear part of vehicle rotates in small angle anticlockwise, the vehicle body moves in parallel with the deformed guardrail. No lateral rotation occurs.	The vehicle body bounces back from the guardrail with rotation and parallel movements at high Speed. Lateral rotation occurs.

The analysis of collision between vehicles and guardrails suggests that the vehicle should not be simplified as a particle of mass in modeling. This is because that the clockwise rotation of the vehicle can easily lead to the secondary crashes. It was reported that lateral rotations happen to more than 75% small vehicles after colliding with guardrails (Ray et al., 1986). When the angular acceleration  $\alpha$  exceeds a certain level, the vehicle rotates at a high velocity which can directly throw the drivers and occupants out of the vehicle. That is main reason why the fatality rate is high for collisions between the small vehicles and the wave beam guardrails.

### 3. A mechanical model of collision between car and guardrail without lateral rotation

A large number of experimental studies show that, as long as the guardrail has strong enough anti-collision performance, the motion state of the guardrail will be basically parallel to that of the guardrail after a large vehicle hits the guardrail and a small car impacts the guardrail at a small angle. This is also one of the important functions of the guardrail-the guiding role. The motion status of the vehicles in t2 Stage without lateral rotation is shown in Figure 2.

Let denote the variables as:

$\Delta S$  - lateral displacement of the vehicle (m);

$C$  - distance between vehicle's center of gravity and its front bumper (m);

$\theta$  - collision angle of the vehicle;

$b$  - width of vehicle (m);

$z$  - parameter of guardrail deformation.  $z \approx 0$  for concrete guardrail;

$z = 0.3 \sim 0.6$  for wave-shape guardrail;

$y_1, y_2, y_3$  - the coordinate values of vehicle's center of gravity at different stages.

Assume that the lateral velocity component of the stage t2 is 0 and that the change in speed and direction does not lead to vehicle rollover, the lateral displacement of the vehicle's center of gravity in the y direction,  $\Delta S$ , can be expressed in the following formula:

$$\Delta S = y_1 - y_3 = c \sin \theta - b_2 (1 - \cos \theta) + Z_1 \quad (5)$$

where:

$$y_1 = c \sin \theta + \frac{b}{2} \cos \theta; y_1 = \frac{b}{2} - z .$$

The impact force  $F$  of the guardrail and the level of the lateral deformation follow a linear relation, that is:

$$\frac{d^2 y}{dt^2} = -\sqrt{\frac{k}{m}} y \quad (6)$$

where  $k$  is the elastic modulus of the guardrail and  $m$  is the mass of the vehicle.

The differential equation (6) can be solved as:

$$y = z \cdot \cos \left( \omega \cdot t + \frac{\pi}{2} \right) \quad (7)$$

Thus, the lateral velocity of vehicle's center of gravity,  $v_y$ , and time  $t$  follow a relation of sine curve.

The average speed ( $\bar{v}_y$ ) and the maximum speed ( $v_{y \max}$ ) satisfy:

$$\bar{v}_y = \frac{2}{\pi} \cdot v_{y \max} = \frac{2v_0 \sin \theta}{\pi}$$

The amount of time used for the lateral displacement ( $\Delta t$ ) is:

$$\Delta t = \frac{\Delta S}{\bar{v}_y} = \frac{\pi \Delta S}{2v_0 \sin \theta}$$

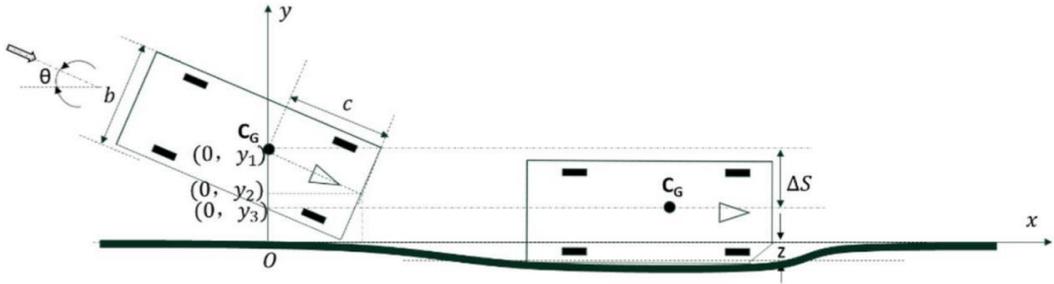


Fig. 2. Model of vehicle collision with the guardrail without lateral rotation

According to the theorem of momentum, the average impact force  $\bar{F}$  can be expressed as:

$$\bar{F} = m \cdot \frac{v_0 \sin \theta}{\Delta t} = \frac{2m(v_0 \sin \theta)^2}{\pi [c \sin \theta - \frac{b}{2}(1 - \cos \theta) + z]} \quad (8)$$

Since the average velocity ( $\bar{a}_y$ ) and the maximum velocity ( $a_{ymax}$ ) satisfy:

$$\bar{a}_y = \frac{2}{\pi} \cdot a_{ymax}$$

the maximum impact force  $F_{max}$  can be calculated as:

$$F_{max} = \frac{\pi}{2} \cdot \bar{F} = \frac{m(v_0 \sin \theta)^2}{c \sin \theta - \frac{b}{2}(1 - \cos \theta) + z} \quad (9)$$

Equation (9) is used to calculate the collision force of the vehicle without lateral rotation. Once the lateral rotation occurs, the collision motion is more complicated, even before transverse rotation occurs in the vehicle collision, this model is still suitable. The causes and prevention of lateral rotation will be studied separately. The calculation model mainly considers the factors such as vehicle mass, vehicle size, collision angle and collision velocity, etc. The impact area is mainly determined by the width of the fences and has little relationship with the vehicle model, so the impact area is not considered in this paper.

We now compare the results from the above Equation (9) with those from the classic mechanical mode in Equation (1). In this analysis, we set the vehicle

velocity as 96km/h, the collision angle as 15°, the 31 collision of rigid guardrail ( $z=0$ ). (Note: Why the angle at which the vehicle was chosen to collide the guardrail was 15° and the collision velocity was 96 km/h, the reasons are as follows: first, this data is derived from the data provided by the people's Republic of China Industry Recommendation Standard (JTG/T D—2006), which started in April 2000 and ended in April 2001, for which six batches of personnel have been organized to investigate 33 highways of more than 7,000 kilometers in 16 provinces and cities, with effective data on nearly 1,000 collision accidents obtained, more than 400 crash sites surveyed, and nearly 1,000 photos and some video materials taken, the result was the average collision angle is 15°. The other evidence is that two European and American experts, Bloom and Bush, have carried out actual vehicle collision experiments, using the 15° collision angle and the collision velocity of 96 km/h). The results are shown in Table 2.

From Table 2, it is shown that for small vehicles, the calculated impact force  $F_{max}$  from the classic model is consistently larger than  $F_{max}^0$  from the classic model (Eq. 1). For smaller vehicles, these two estimated values are relatively close. The gap increases when the mass of vehicle increases. For the large and heavy vehicles,  $F_{max}$  is almost 20% over  $F_{max}^0$ . The lateral impact force of the vehicle on the guardrail not only depends on the lateral kinetic energy, but also be directly related to vehicle's center of gravity and the length of axle. This explains that there are differences in the maximum lateral impact force in the real vehicle crash tests. As there are limits for the conditions for real vehicle experiments, so this paper is mainly theoretical research with the help of domestic and foreign experimental data for comparative analysis.

Table 2. A Comparison of  $F_{max}$  and  $F_{max}^0$

Mass Vehicle (t)	distance between vehicle center of gravity and its front bumper (value of c, unit: m)	Vehicle width b (m)	$F_{max}$ calculated from Eq. (9) (KN)	$F_{max}^0$ calculated from Eq. (1) (KN)	Difference as Percentage
2.0	1.0	1.60	410.7	365.6	11.0%
2.0	1.3	1.65	308.4	266.6	13.6%
2.0	1.6	1.70	247.0	209.8	15.1%
2.0	1.9	1.75	205.9	172.9	16.0%
5.0	2.2	1.80	441.4	367.7	16.7%
5.0	2.5	1.85	386.3	319.9	17.2%
5.0	2.8	1.90	343.5	283.1	17.6%
5.0	3.1	1.95	309.2	253.9	17.9%
10.0	3.4	2.00	562.2	460.3	18.1%
10.0	3.7	2.05	515.5	420.9	18.3%
10.0	4.0	2.10	475.9	387.7	18.5%
10.0	4.3	2.15	441.9	359.4	18.7%
20.0	4.6	2.20	825.0	669.9	18.8%
20.0	4.9	2.25	773.4	627.2	18.9%
20.0	5.2	2.30	728.0	589.6	19.0%
20.0	5.5	2.35	687.6	556.3	19.1%
30.0	5.8	2.40	977.1	789.8	19.2%
30.0	6.1	2.45	928.3	749.7	19.2%
30.0	6.4	2.50	884.2	713.5	19.3%
30.0	6.7	2.55	844.0	680.6	19.4%

**4. Model validation with the data from the real vehicle crash testing**

We used the data from the real vehicle crash testing to validate the developed model. There are two sets of well-known guardrail testing performed in the United States. The first testing (referred as Testing One) was performed by Buth in 1984 and the second testing (referred as Testing Two) was conducted by Bronstad et al. (1988). For comparison purpose, we set the collision velocity as 96km/s and the collision angle as 15°, and compare the theoretical prediction values of formula (Eq. 9) and (Eq. 1) with those of Bronstad and Buth’ results. The results are presented in Table 3. The analysis of results has shown that the improved mathematical model of collision without lateral rotation is more accurate than the classic model.

The classic model was derived with assumption that the relationship between speed and time is a linear function. In reality, the relationship approximately follows a sinusoidal curve rather than a linear relation. This is the reason why the predicted value of impact force is always smaller than the values from real vehicle crash testing.

Table 3. Model validation with the data from the real vehicle crash testing

Vehicle Mass (t)	Maximum Lateral Impact Force (KN)				Values of vehicle parameters (m)
			Results from Testing One	Results from Testing Two	
	$F_{max}$	$F_{max}^0$			
2.043	133.9	129.0	124.5	133.4	c=2.55, b=2
9.080	304.0	244.7	373.6	311.4	c=5.62, b=2
18.16	504.5	405.2	667.2	667.2	c=6.76, b=2.2
31.78	1111.3	897.8		1112	c=5.4, b=2.2

**5. Conclusions**

The movement of vehicles after colliding with high-way guardrails is affected by vehicle type, collision speed, collision angle, guardrail type, and road friction coefficient. At present, the deformation buffering function has been more intensified to protect vehicles and the life of drivers in the guardrail design process. But a large number of traffic accidents show that concrete guardrail is safer than corrugated beam guardrail (as mentioned in this article). However, currently the passive safety technologies of the

automobile have become more and more mature, so it is unnecessary to lay too much emphasis on the deformation. Lateral rotation does not occur when large and heavy vehicles collide with any type of guardrails or when any vehicle collides with the concrete guardrails. In this paper, an improved mechanical model of vehicle collision with highway guardrail is developed. The improved model has been compared with the classic model and was validated with the data collected from the real vehicle crash testing. The numerical analysis has shown that the new model can achieve higher accuracy in predicting the impact force. The improved model can provide practical guidelines for highway guardrail design. Future research along this direction will enhance the model with consideration of other factors including the effects of vehicle's lateral rotation, friction with ground, and the energy loss caused by the lateral wobble of the center of gravity of the vehicle.

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