

SCIENTIFIC APPLICATIONS OF THE AHP METHOD IN TRANSPORT PROBLEMS

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Abstract: *In methodologies of solving different transport problems where the best decision has to be determined usually a number of chosen quantitative criteria are incorporated, which describe qualitative parameters of transport systems in quantitative terms. Often weights which reveal importance of such criteria must be evaluated. The realm of proprietary methods used in engineering sampling and experimental studies does not comprise methods of weight evaluation. Consequently, expert evaluation methods, which elicit weights of criteria from experienced, qualified and fair experts, are used. Among the most popular such methods is the method AHP (Analytic Hierarchy Process). Scientists of Vilnius Gediminas Technical University used this method for investigation of interrelationship of elements of a transport system; for evaluation of influence of the interrelationship on road traffic safety; and for evaluation of quality of passenger railway transportation service.*

Key words: *AHP method, transport system, weight of criteria*

1. Introduction

In methodologies of solving different transport problems usually a number of chosen quantitative criteria are incorporated, which describe qualitative parameters of transport systems in quantitative terms. Often weights which reveal importance of such criteria must be evaluated. The realm of proprietary methods used in engineering sampling and experimental studies does not comprise methods of weight evaluation. Consequently, expert evaluation methods, which elicit weights of criteria from experienced, qualified and fair experts, are used. Among the most popular such methods is the method AHP (Analytic Hierarchy Process) [2], [9], [14]. Scientists of Vilnius Gediminas Technical University used this method for investigation of interrelationship of elements of a transport system; for evaluation of influence of the interrelationship on road traffic safety; and for

evaluation of quality of passenger railway transportation service [7], [10].

The AHP is a powerful tool of multi-criteria decision-making developed by Saaty (1980). The AHP is used for solving complex decision-making problems in different areas (e.g. civil engineering, transport, social and economic development, project selection and material science). Basically, the AHP is designed in such a way that it reduces complexity of decision-making problems to a series of one-on-one comparisons, following synthesis of results of such comparisons. However, calculation takes is rather cumbersome and is requiring high researcher's skills and there may be a lack of transparency in the whole decision-making process. Furthermore, disadvantages of the AHP are associated with imposed limitation of the 9-point scale of evaluation [13]. The scale is obviously insufficient in the case of a large number of criteria. Furthermore, it does not allow

a decision-maker to assign the zero weight to an insignificant criterion.

In methodologies used in this paper for solving transport problems the method AHP of pairwise comparison developed by T. Saaty AHP was used [12]. The method is designed to help researchers to estimate weights of significance of criteria. At the initial step of the method the pairwise comparison matrix $\mathbf{P} = \|p_{ij}\|$ ($i, j = 1, 2, \dots, m$), where m is the number of criteria, is formed from information elicited from each expert participating in the research. The experts compare all criteria R_i and R_j with each other evaluating levels of their comparative significance. Presented below pairwise comparison matrix represents relationship between unknown weights ω_i of criteria (interactions) only in the ideal case:

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1m} \\ p_{21} & p_{22} & \dots & p_{2m} \\ \vdots & \ddots & & \vdots \\ p_{m1} & p_{m2} & \dots & p_{mm} \end{pmatrix} = \begin{pmatrix} \frac{\omega_1}{\omega_1} & \frac{\omega_1}{\omega_2} & \dots & \frac{\omega_1}{\omega_m} \\ \frac{\omega_2}{\omega_1} & \frac{\omega_2}{\omega_2} & \dots & \frac{\omega_2}{\omega_m} \\ \vdots & \ddots & & \vdots \\ \frac{\omega_m}{\omega_1} & \frac{\omega_m}{\omega_2} & \dots & \frac{\omega_m}{\omega_m} \end{pmatrix} \quad (1)$$

Each element of the matrix indicates relative significance of considered criteria in terms of the evaluated object. The matrix is inverse symmetrical, i.e. $p_{ij} = 1/p_{ji}$. The evaluation scale of 1-3-5-7-9 is used in the AHP technique [12]. Intermediate even numbers may also be used.

Weights obtained by using Saaty's AHP method are derived from the eigenvector, which corresponds to the largest eigenvalue λ_{\max} of matrix \mathbf{P} by its normalization. Final weights are co-ordinates of the normalized vector of weights ω :

$$\mathbf{P} \cdot \omega = \lambda_{\max} \cdot \omega \quad (2)$$

The largest eigenvalue of the matrix \mathbf{P} and the eigenvector may be calculated using computer programs or one of the suggested algorithms, which simplifies finding approximate weights even without using a computer [9], [13], [15].

In using AHP technique, each expert fills in a pairwise comparison matrix-form. An example of such matrix-form filled in by one of the experts (the first one) is presented in Table 2. The experts

performed the pairwise comparison in all possible pairs of criteria A, B, C, ..., I between each other displayed in Table 1, taking into account their influence on traffic safety.

When filling in the questionnaire, each expert should attempt to examine the form if there are any apparent logical contradictions (errors) in the estimates made.

We observe that if the scale of evaluation were not restricted to the set of integers 1-3-5-7-9, and rational numbers could be used instead of integers it would be sufficient to fill in only one row of the matrix \mathbf{P} . Then the filling of whole matrix would be straightforward using the mentioned property of proportionality of its elements [9]. However, Saaty developed a methodology allowing to verify or eliminate contradictions or errors found in filling in the rows or columns of the matrix elements.

Following the rules given below can help an expert with filling in the questionnaire (matrix) of criterion comparison and reducing inconsistency (discordance degree):

- 1) First, the criteria should be ranked according to their significance for the purpose of evaluation. The most significant criterion is assigned the highest rank, the second most significant is given rank 2, etc. while the least important criterion – rank m , where m is the number of the criteria chosen.
- 2) The criteria are written down in the evaluation table (matrix) in the order of their significance according to the ranks obtained.
- 3) All the elements in the 1st row will therefore be smaller than 1, because the 1-st criterion is the most important. All the elements of the matrix above the main diagonal will larger than 1 or sometimes equal to 1 because each criterion is more important (or is of the same importance) than any criterion below.
- 4) The 2-nd, 3-rd and other criteria are compared with the remaining criteria. All the elements of the matrix above the main diagonal will again be not be smaller than 1 because the corresponding criterion is more important than the ones below it.
- 5) None of the elements in the 2-nd row can be larger than the largest element in the 1st row, because the 1st criterion is the most significant while the elements of the matrix

(in an ideal case) represent the relationships between the unknown weights of the criteria. Neither element of the 3-rd row cannot be larger than the largest element in the 2-nd row either. This rule applies to all subsequent rows.

The most significant elements as well as the order of their significances may vary, because each expert refers to his/her personal experience, knowledge, views, etc., assigns weights (significances), determines the relationships between the weights of criteria in accordance with his or her own point of view.

The concordance (consistency) degree of particular estimates of each expert is determined by consistency index *C.I.* and concordance ratio *C.R.* [13].

The consistency index is defined as the ratio [12] as follows:

$$C.I. = \frac{\lambda_{\max} - m}{m - 1} \quad (3)$$

where: *m* is the matrix order and the number of the criteria compared.

In practice, the level of the consistency of matrix **P** may be determined if we compare calculated consistency index *C.I.* in the evaluation matrix with randomly generated (in terms of the scale 1-3-5-7-9 used in AHP method) index *R.I.* found in the same row of the inversely symmetric matrix

[13]. The ratio of consistency index *C.I.* calculated in a particular matrix to the mean value of random index *R.I.* is referred to as consistency ratio *C.R.* showing the degree of matrix consistency (*C.R.* < 0.1):

$$C.R. = \frac{C.I.}{R.I.} \quad (4)$$

2. Evaluation of Interaction between Elements of a Transport System

The original model of the interaction between the physical elements of the TS (transport system) representing 6 interaction levels was proposed in [15]. The first level represents self-interaction between the TS elements, the second – the interaction between the elements, the third – the interaction between the TS elements and the external environment, the fourth – the interaction between various transport modes, the fifth – the interaction of the TS with national economic and non-production sectors and the sixth – the impact of the TS on the Gross Added Value (GAV). To determine significance of transportation parameters at various interaction levels, the AHP method was suggested.

In the paper by researchers of Vilnius Technical University [10] the first three interaction levels of the TS elements influencing the accident rate on the roads and highways were analysed (Table 1).

Table 1. A list of criteria describing the interaction between the elements at various levels of the TS having an impact on the accident rate on the road

Interaction No.	Interaction code	A name and detailed description of the interaction
1	I-1-1 (A)	The interaction between a traffic participant (freight) and a traffic participant (freight)
2	I-2-2 (B)	The interaction between a motor vehicle and a motor vehicle
3	I-3-3 (C)	The interaction between a motor road and a motor road (and its elements)
4	II-1-2 (D)	The interaction between a traffic participant (freight) and a vehicle
5	II-1-3 (E)	The interaction between a traffic participant (freight) and a motor road (and its elements)
6	II-2-3 (F)	The interaction between a vehicle and a motor road (and its elements)
7	III-1-E (G)	The interaction between a traffic participant (freight) and the environment
8	III-2-E (H)	The interaction between a vehicle and the environment
9	III-3-E (I)	The interaction between the motor road (or its elements) and the environment

Table 2. Data on comparing the criteria elicited from an expert

Criterion	A	B	C	D	E	F	G	H	I	Weights ω_i	Ranks e_i
A	1	1/3	7	1/3	2	3	4	5	7	0.144	3
B	3	1	8	2	4	5	6	7	9	0.302	1
C	1/7	1/8	1	1/9	1/6	1/3	1/3	1/2	2	0.023	8
D	3	1/2	9	1	4	5	7	8	9	0.268	2
E	1/2	1/4	6	1/4	1	2	3	5	8	0.109	4
F	1/3	1/5	3	1/5	1/2	1	1	2	3	0.054	5
G	1/4	1/6	3	1/7	1/3	1	1	2	3	0.048	6
H	1/5	1/7	2	1/8	1/5	1/2	1/2	1	2	0.031	7
I	1/7	1/9	1/2	1/9	1/8	1/3	1/3	1/2	1	0.019	9

All the experts [16], taking part in research were given a description of the levels of the interaction of the TS elements (Table 1). They analysed the situation, then filled in the matrix of pairwise comparison according to AHP requirements and determined the impact of interaction in terms of the accident rate.

All the elements in the second row (B) of Table 2 should be, theoretically, three times as large as the respective 1st (A) row elements, whereas the 3-rd (C) row elements should be smaller than the respective 1-st (A) row elements (theoretically 7 times), which would imply that contradictions are absolutely absent. In our case, acceptable absence of contradictions can be observed. There are only few logical contradictions in other rows of the matrix-form filled in by this expert either.

The matrix is consistent if consistency ratio $C.R.$ is smaller than 0.1 [12]. For the data presented in Table 2, the consistency index of the comparison matrix of the 1st expert $C.I. = 0.050$ and consistency ratio $C.R. = 0.034 < 0.1$.

Final weights obtained from elicited evaluations by the first expert are presented in Table 2 as well.

3. Estimates of Weights a Group of Experts and Their Consistency

To obtain the final weights of criteria, the matrix-forms, including estimates provided by experts and containing no serious errors, and with calculated consistency ratio $C.R.$ smaller than 0.1 were selected. Consistency index $C.I.$, consistency ratio $C.R.$ and the weights of criteria ω_i of expert estimates were determined by applying the AHP method.

The reliability of the results is expected to be much higher making the evaluation by a groups of experts and then taking the average aggregate weight for each criterion. In our case a group consisting of 16 experts, transport engineering specialists, was participating. Calculation of consistency of results of the estimates within a group is based on ranking of criteria, as was estimated by each expert, and is quantitatively gauged using concordance coefficient W [4]. It is plausible to mention here that, in contrast, the AHP method is used only for defining the consistency of estimates provided by each particular expert. To specify the consistency of the evaluation results of the whole group of the experts the algorithm proposed by [8] was used. First, weights of each criterion ω_i ($i = 1, 2, \dots, m$) are calculated and, then, the ranks of these criteria are determined. Weights of criteria ω_{ki} calculated for all experts are presented in Table 3 (k is the number of experts; $k = 1, 2, \dots, q$). Ranks of criteria as was evaluated by 16 experts are displayed in Table 4.

Concordance coefficient W is calculated by the equation [4]:

$$W = \frac{12 \cdot S}{q^2 \cdot m \cdot (m^2 - 1)} \tag{5}$$

where: m is the number of criteria; q is the number of experts; S is the sum of the squares of deviations from the sum of ranks

$$e_i = \sum_{k=1}^q e_{ik} \tag{6}$$

of rank of each criterion (Table 4) from the mean value of ranks

$$\bar{e} = \frac{\sum_{i=1}^m e_i}{m} \tag{7}$$

calculated by the following formula:

$$S = \sum_{i=1}^m (e_i - \bar{e})^2 \tag{8}$$

Table 3. Weights ω_{ki} calculated for evaluation criteria by the group of experts

Expert	Criterion								
	A	B	C	D	E	F	G	H	I
E1	0.144	0.302	0.023	0.268	0.109	0.054	0.048	0.031	0.019
E2	0.014	0.285	0.070	0.214	0.061	0.168	0.036	0.134	0.019
E3	0.104	0.326	0.154	0.224	0.047	0.076	0.031	0.022	0.016
E4	0.217	0.162	0.029	0.329	0.073	0.110	0.048	0.018	0.014
E5	0.268	0.134	0.062	0.268	0.030	0.132	0.030	0.062	0.017
E6	0.211	0.165	0.015	0.325	0.021	0.029	0.107	0.063	0.064
E7	0.263	0.122	0.055	0.308	0.130	0.058	0.019	0.027	0.018
E8	0.156	0.326	0.073	0.110	0.046	0.223	0.021	0.015	0.031
E9	0.121	0.074	0.040	0.229	0.030	0.182	0.016	0.217	0.021
E10	0.115	0.070	0.043	0.315	0.029	0.181	0.016	0.212	0.019
E11	0.213	0.105	0.161	0.329	0.050	0.072	0.022	0.032	0.016
E12	0.074	0.222	0.025	0.312	0.035	0.155	0.051	0.108	0.018
E13	0.266	0.131	0.061	0.266	0.034	0.131	0.034	0.059	0.017
E14	0.218	0.163	0.036	0.314	0.080	0.100	0.048	0.019	0.022
E15	0.315	0.212	0.181	0.115	0.069	0.043	0.029	0.019	0.016
E16	0.228	0.186	0.020	0.304	0.110	0.044	0.029	0.064	0.015
Average weight $\bar{\omega}_i$	0.183	0.187	0.066	0.264	0.060	0.110	0.037	0.069	0.021
Rank	3	2	6	1	7	4	8	5	9

Table 4. The ranks of criteria e_{ik} as was estimated by the experts

Criterion	Expert																Sum	Rank e_i
	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16		
A	3	9	4	2	1.5	2	2	3	4	4	2	5	1.5	2	1	2	48	3
B	1	1	1	3	4	3	4	1	5	5	4	2	3.5	3	2	3	45.5	2
C	8	5	3	7	5.5	9	6	5	6	6	3	8	5	7	3	8	94.5	6
D	2	2	2	1	1.5	1	1	4	1	1	1	1	1.5	1	4	1	26	1
E	4	6	6	5	7.5	8	3	6	7	7	6	7	7.5	5	5	4	94	5
F	5	3	5	4	3	7	5	2	3	3	5	3	3.5	4	6	6	67.5	4
G	6	7	7	6	7.5	4	8	8	9	9	8	6	7.5	6	7	7	113	8
H	7	4	8	8	5.5	6	7	9	2	2	7	4	6	9	8	5	97.5	7
I	9	8	9	9	9	5	9	7	8	8	9	9	9	8	9	9	134	9

Concordance coefficient in our case appeared to be $W = 0.651$.

Significance of the obtained coefficient of the concordance can be estimated using the criterion χ^2 [4]:

$$\chi^2 = W \cdot q \cdot (m - 1) = \frac{12 \cdot S}{q \cdot m \cdot (m + 1)} \quad (9)$$

If χ^2 calculated by equation (9) is larger than critical $\chi_{\alpha, \nu}^2$ obtained from the table of χ^2 distribution with a degree of freedom $\nu = m - 1$ and significance level α chosen to be either 0.01 or 0.05, then the estimates elicited from the experts are considered to be consistent.

In this particular case (when $S = 10004$; $q = 16$; $m = 9$ and $W = 0.651$), $\chi^2 = 83.37$, while critical value $\chi_{\alpha, \nu}^2$ obtained from the table of chi-square distribution with a degree of freedom $\nu = m - 1 = 8$ and significance level $\alpha = 0.05$ is equal to $\chi_{\alpha, \nu}^2 = 15.51$. Hence, the estimates of the experts are consistent ($83.37 \gg 15.51$).

A-priori evaluation of traffic accident rate is as important as complicated. Evaluation depends on various technical parameters: performance of vehicles, different properties and characteristics of road elements and pavement, behaviour of traffic participants, types of freight, climate and

weather conditions, traffic flows, and other factors. Accidents take place during interactions between moving vehicles, traffic participants, goods, and other surrounding elements near roads. Parameters incorporated into analysis differ considerably. Such elements highly interact with a Transport System (TS). In particular, it should be noted that parameters of material elements in particular roads considerably differ. Even in the case of multidimensional and considerably different parameters it was possible to estimate levels of interaction of elements of TS in terms of traffic accident rate by using the method AHP. The major part of the investigation, estimation of weights of criteria, is presented in the paper. Weights of significance of 9 different types of interaction between elements of TS based on expert estimates were estimated. The weights obtained from 16 matrix-forms filled in by experts are consistent because the calculated

consistency ratio $C.R.$ is lower than 0.1 (ranging from 0.033 to 0.098) in all pairwise comparison matrices.

4. Evaluation of quality of railway passenger transportation service

Quality of travel by train (QTT) is described by both qualitative and quantitative criteria. Quantitative evaluation of quality of railway passenger transportation service expressed by a single number is an easy-to-comprehend way of revealing results of evaluation, which is a case of using multiple criteria methodologies of quantitative evaluation, and prominence of such methods [6], [7], [16]. Using such methodologies implies that weights of chosen evaluation criteria are determined. Significance of particular criteria may differ. In [5] 49 QTT criteria were chosen, which are distributed between four various groups A, B, C, D (Fig. 1).

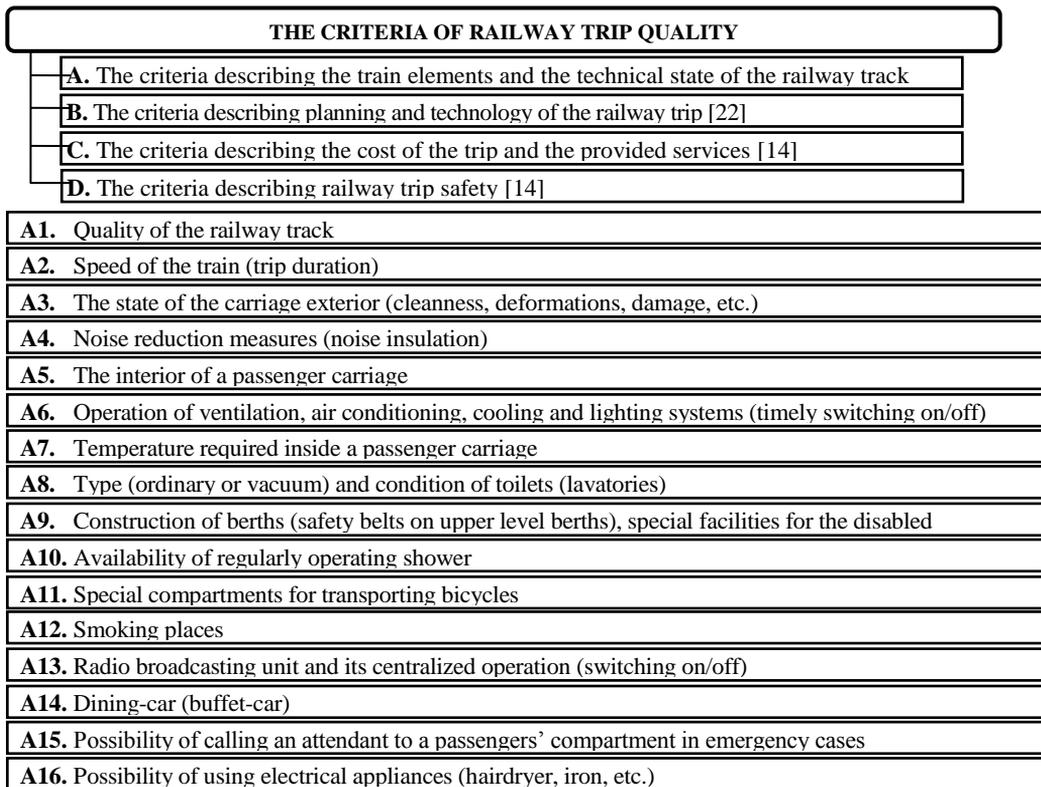


Fig. 1. Criteria describing the railway trip quality

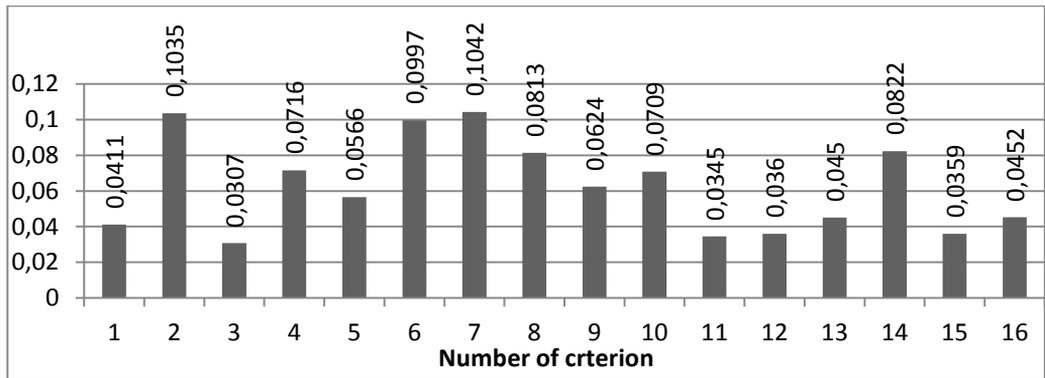


Fig. 2. Mean weight values of sub-criteria of group A, determined by AHP method

Average weights of the criteria were determined by using the expert evaluation method AHP. In group A of sub-criteria describing train elements and technical state of rails (railway track), consisting 16 sub-criteria, weights were obtained by surveying experts, while weights in three remaining categories B, C, and D were estimated using opinions of other groups of respondents, respectively: passengers (P); service staff of the train (ST); and the administration staff (AS) of the joint-stock company ‘Lithuanian Railways’ (AB „Lietuvos geležinkeliai“ – “LG”). Weights of sub-criteria of the group A were obtained using AHP method; the weights are presented in Fig. 2. Weights of criteria of other three groups B, C, D were estimated as well using AHP method [7].

5. Conclusions

In order to solve such problems where the best decision has to be determined usually a number of chosen quantitative criteria are incorporated, which describe qualitative parameters. Methodologies of evaluation of available solutions imply using weights of importance of such criteria. Among the most popular such methods of determining weights of criteria the method AHP (Analytic Hierarchy Process) is found.

Many problems related to transport systems could be solved using the above-mentioned methodologies. In the paper two cases of using the AHP method for solving transport problems are described. The first problem evaluates interaction between elements of a Transport System in terms of traffic safety. The second

problem is devoted to evaluation of quality of railway passenger transportation service.

For every matrix-form with elicited from an expert weight estimations consistency index *C.I.* and concordance ratio *C.R.* were calculated. Such forms of unacceptably large indexes were discarded, while the forms with acceptable indexes were used.

Concordance of opinions of the whole group of experts was performed using the theory of concordance by Kendall. Final weights of criteria were calculated as averages of obtained weights from all the experts.

Practical use of the AHP method revealed its effectiveness for solving transport problems. Obtained weights of criteria allow to use multiple criteria decision-making methods [1][3][11] (in order to evaluate available alternatives in terms of objectives of transport problems, for example to evaluate quality of railway passenger transportation service in different chosen trains, level of safety of traffic in different cities or districts, etc.

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